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INTERSTELLAR HYDROGEN AND THE LOCAL SYSTEM

By David S. Heeschen and A. Edward Lilley

HARVARD COLLEGE OBSERVATORY

Communicated by D. H. Menzel, August 11, 1954

In two regional studies now under way at the George R. Agassiz Station of Harvard Observatory, we have concentrated on observations of the 21-cm. line of hydrogen for the sections of the galactic center and anticenter.[1, 2] The general program of 21-cm.-line research and the equipment in use have been described by Bok and Ewen.[3]

An interesting feature of galactic structure near the sun is revealed by the combined observations of the center and anticenter surveys. The radio measures for the galactic center and anticenter taken, respectively, at galactic longitudes 327° and 147° lie on a circle which is normal to the galactic plane. The observations are presented here in polar form, with galactic latitude the angular co-ordinate and hydrogen-line peak brightness temperature the radial co-ordinate.

All observations were made with the antenna tracking a fixed position in the sky. The receiver, as it is tuned in frequency, continuously records the brightness temperature in the hydrogen line as a function of frequency. The maximum value recorded for the brightness temperature is referred to as the "peak brightness temperature," and its value is shown in the diagram. Four or more frequency scans were made at each position. The probable error of a peak brightness temperature, from the mean of four scans, is about ±5° K.

The diagram shows two distinct features of brightness distribution. The brightness temperature reaches its highest values in the plane and drops off rapidly with latitude, as is to be expected. In addition, however, there is a second concentration of hydrogen along a line inclined at about 20° to the galactic plane.

It was first pointed out by John Herschel[4] that the plane of symmetry for the brightest stars was inclined 20° to the galactic equator. This phenomenon, known as Gould's Belt, has been studied by many investigators; Bok has summarized their results[5] and lists bright B, Be, and A stars and extended dark nebulae as belonging to Gould's Belt. Recently, Nassau and Morgan have shown[6] that the OB stars within 400 parsecs of the sun show a similar tendency.

It now appears that the near-by clouds of neutral hydrogen are also concentrated to Gould's Belt. This is not surprising, in view of the close association of gas and dust recently pointed out by one of us.[2] The regions of high intensity of the 21-cm. line of neutral hydrogen at latitudes +20° and −20°, shown in Figure 1, coincide with regions of heavy obscuration. These regions of high peak brightness

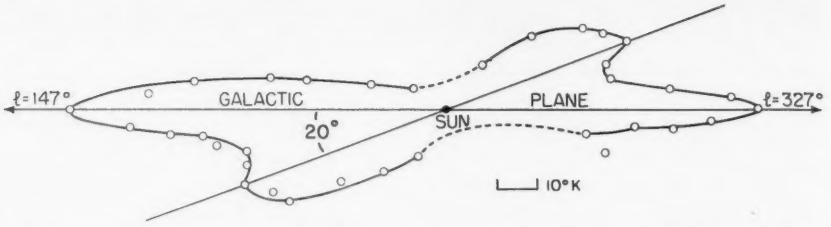

FIG. 1.—Distribution of the peak brightness temperature with galactic latitude for a section perpendicular to the galactic plane at longitudes 327° and 147°.

temperature correspond well to the position of Gould's Belt at these longitudes and lend additional support to the hypothesis of a "local system" or give evidence for some definite feature of local structure in the neighborhood of the sun. It appears that the neutral hydrogen gas, as well as the dust and early-type stars, is a constituent of the local system, which means that a good fraction of all matter near the sun is concentrated toward Gould's Belt.

This work was supported in part by grants from the National Science Foundation and by a gift from a friend of the George R. Agassiz Station.

[1] D. S. Heeschen, "Investigations of the 21 cm. Line in the Section of the Galactic Center" (paper presented at the meeting of the American Astronomical Society, Ann Arbor, Michigan, 1954), *Astron. J.* (in press [abstr.]).

[2] A. E. Lilley, "21 cm. Analysis of the Taurus Dark Nebulae Complex" (paper presented at the meeting of the American Astronomical Society, Ann Arbor, Michigan, 1954), *Astron. J.* (in press [abstr.]).

[3] B. J. Bok and H. I. Ewen, "21 cm. Research at Agassiz Station" (paper presented at the meeting of the American Astronomical Society, Ann Arbor, Michigan, 1954), *Astron. J.* (in press [abstr.]).

[4] J. F. W. Herschel, *Results of Astronomical Observations at the Cape of Good Hope* (London, 1847), p. 385.

[5] B. J. Bok, *The Distribution of Stars in Space* (Chicago: University of Chicago Press, 1937), p. 97.

[6] J. J. Nassau and W. W. Morgan, *Pub. Observatory Univ. Mich.*, **10**, 49, 1951.

ON THE ORIGIN OF THE LUNAR SURFACE FEATURES

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Communicated October 29, 1954

1. *Introduction.*—Much has been written on the surface features of the moon and their possible interpretations. Two main types of explanation have been advanced: volcanic origin or origin by impact of meteorites. Recent advocates of the former hypothesis are Goodacre,[1] Spurr,[2] and Escher;[3] of the latter, Daly,[4] Baldwin,[5] and Urey.[6]* An examination of the arguments used in these studies shows that the same empirical facts are sometimes used to derive opposite con-

*Jeffreys (*The Earth*, p. 353) appears to favor the volcanic hypothesis.

clusions. Some of this ambiguity can undoubtedly be removed by increased rigor, and, in fact, some of the views held by proponents of the volcanic hypothesis have been convincingly dismissed by Baldwin and Urey. It seemed to the writer that an independent study, based not on published records but on new visual observations with a large telescope, might resolve some of the ambiguities. Accordingly, a systematic study was begun with the 82-inch telescope about a year ago, with a specially constructed binocular attachment carrying two wide-angle eyepieces. This allows continuous observation for several hours without eyestrain and leads to increased visibility of detail. The magnification is 900×, and with good seeing the resolving power exceeds appreciably that of the best photographs. Crater Ptolemy was used as a test object; under good conditions over one hundred small craters can be seen on its floor. The study is not complete, but some preliminary results and conclusions are mentioned here. A fuller account, together with more detailed references to the literature, will be published at a later time.

It will appear that the impact hypothesis is confirmed as the chief cause of lunar topography. But the source of the "planetesimals" is found to be neither meteorites nor small masses of the type that formed the moon by accretion. Furthermore, evidence is presented that the moon was nearly completely melted by its own radioactivity, some 0.5 to 1 billion years after its formation, and that the maria were formed during this epoch, and that they were not, as has been supposed, primarily the result of melting caused by the impacts themselves.

Because of the relevance of certain dynamical and physical considerations for the interpretation of the observed surface features, these results are mentioned first.

2. *Dynamical Considerations.*—Observations of the lunar rotation, interpreted on the basis of the theory developed largely by Lagrange, Laplace, and Poisson and summarized in two standard textbooks,[7, 8] yield important information on the moments of inertia of the moon. As is well known to the specialist, the observations of the lunar rotation are to a remarkable degree of precision contained in Cassini's three empirical laws:[8]

1. The Moon rotates uniformly about an axis which is fixed with respect to the Moon itself. The period of this rotation is identical with the sidereal period of the Moon in its orbit, namely 27.321661 days.

2. The pole of the lunar rotation z makes a constant angle (1°35′) with the pole of the ecliptic Z, which may here be regarded as a fixed point on the celestial sphere.

3. In consequence of the nearly uniform regression of the lunar node on the plane of the ecliptic and the nearly constant inclination of the lunar orbit (5°9′), the pole of the Moon's orbit P is known to describe a small circle about Z in a period of $18^2/_3$ years. The arc of a great circle zP contains also the pole Z. In other words, the planes of the lunar orbit and the lunar equator intersect on the ecliptic, the latter plane being intermediate between the two former.

If, for simplicity, one thinks of the moon as a homogeneous, triaxial ellipsoid and takes the axis of rotation as unit length, the axis pointed at the earth may be called $1 + \beta$ and the third axis (in the lunar equator and perpendicular to the line of sight) $1 + \alpha$. Then one finds from the definition of the moment of inertia that the moment C around the polar axis is, to the first order and apart from a constant factor, $1 + \alpha + \beta$; the moment around the axis pointing toward the earth,

$A = 1 + \alpha$; and $B = 1 + \beta$. The ratios determining the motions of the ellipsoid are, then, to the first order,

$$\frac{C - A}{B} = \beta; \qquad \frac{C - B}{A} = \alpha; \qquad \frac{B - A}{C} = \gamma = \beta - \alpha.$$

Now the validity of Cassini's laws is not obvious or completely general. The theory shows that it requires $C > B > A$;* otherwise, the physical librations of the moon would not be stable, or they might show secular terms, contrary to observation. A third requirement is of special interest when the history of the earth-moon system is considered—namely, that $\beta < {}^2/_3\mu$, in which μ is the mean motion of the node of the lunar orbit on the ecliptic, in terms of the mean motion of the moon itself, 0.004019 (the period of the node is 18.9 years). We return to this condition below.

The mean inclination of the lunar equator on the ecliptic, θ_0, has been found from observation to be close to 1°32′, with an uncertainty of about a minute of arc.[9] It is found from theory to depend on the inclination of the moon's orbit, i, and on β and μ, defined before, as follows:[10]

$$\theta_0 \cong \frac{3i\beta}{2\mu + \mu^2 - 3\beta}\left(1 - \frac{1}{2}\mu + \frac{5}{8}\mu^2\right) \cong i\left(\frac{2\mu}{3\beta} - 1\right)^{-1}. \qquad (1)$$

It is seen that, unless β should approach ${}^2/_3\mu = 0.00268$, it can be derived with considerable precision. The numerical value of β found from equation (1) is 0.00062, with an uncertainty of about 1 per cent. This is 17 times larger than the value computed on the assumption that hydrostatic equilibrium exists in the moon to-day,[11] $\beta' = 0.0000375$. This large discrepancy indicates two things: (*a*) the moon may have formed much closer to the earth than it is now, and (*b*) the moon has resisted for some 4×10^9 years making isostatic adjustment; this, incidentally, throws doubt on geophysical hypotheses invoking the assumption of no or small permanent strength in the mantle of the earth.

Much less is known about α or γ. The latter occurs as a factor in both a monthly and an annual term found in the theoretical expression for the physical libration in longitude; but the monthly term is too small to be observed, and the annual term is just within reach and is not known with any precision. Franz found $\gamma = 0.00032$ and Weimer[9] 0.00027 ± 0.00003. It is probable, however, that the uncertainty is greater than is suggested by these figures. Apparently, α and γ are each about ${}^1/_2\beta$. Now, if the moon was molten at one time and solidified with its periods of rotation and revolution synchronized, as at present, then the theoretical ratio of $\alpha/\beta = f$ would be fixed, being ${}^1/_4$.[11] While the observational data quoted perhaps do not entirely exclude the possibility that f is indeed ${}^1/_4$, this would imply rather serious systematic errors in the measures of γ. Weimer's result[9] is $f = 0.57 \pm 0.04$. It is clearly necessary that additional photographic determinations of γ (or f) be made; such work has been started at the Yerkes Observatory.

Curious problems arise when one traces the history of the moon's rotation back in time. One would assume that the moon's bulge pointing toward the earth was

* More generally, it requires $B > A$ and $(C - B)(A - C) < 0$.

formed at some very early epoch, when the moon was much closer to the earth. Since the maintenance of the bulge requires the moon's interior to have considerable strength, the initial bulge cannot have been smaller but may have been larger than the present. Therefore, the initial value of β may have been larger than the present one. But, when the moon was closer, μ must have been smaller. The theory[12] of μ can be used to derive an explicit relation between μ and the ratio a/a_0, where a_0 is the present semimajor axis of the moon's orbit. The result is

$$\mu = -0.00420\left(\frac{a}{a_0}\right)^3 + 0.000117\left(\frac{a}{a_0}\right)^{4\frac{1}{2}} + \ldots \qquad (2)$$

This third-power dependence on a is equally divided between the absolute motion of the node becoming slower as a decreases and the mean motion of the moon becoming faster. Thus μ decreases rapidly with a as one goes back in time, and it will fall below the critical limit of $1.5\beta = 0.00093$ unless the spheroidal shape of the earth should prevent this.

This, however, is not the case. The contribution to μ from the oblateness is

$$\mu' = -\frac{3}{2}\frac{C - A}{Ma^2}, \qquad (3)$$

where M is the mass of the earth and C and A are the moments of inertia. The minus sign means that a regression is caused, as is true of the solar part, equation (2). Thus the two effects are additive. The present value of C is $0.334MR^2$, if R is the radius of the earth, while $(C - A)/C = 1/297$ and $a/R = 60$. The present value of μ' is therefore 5×10^{-7}, or 10^4 times smaller than the solar term, equation (2). While the ratio μ/μ' changes more steeply than a^5 (because the period of rotation of the earth and therefore $C - A$ also depend on a), μ' will be small compared to μ unless $a << 0.4a_0$.

It now follows from equation (2) that μ will reach the critical limit, 1.5β, at $0.608a_0$. One may see what happens as the limit is approached, going back in time. Put $\mu = {}^3/_2(\beta + \epsilon)$ in equation (1); then

$$\theta_0 \cong \frac{i\beta}{\epsilon}. \qquad (4)$$

Therefore, unless i should go to zero simultaneously with ϵ, the obliquity of the lunar equator will increase indefinitely as the limit is approached. The history of i is not well known. G. H. Darwin's work[13] was carried out before the true nature of tidal friction was recognized, and his model calculations with bodily viscosity, which suggest that i has not changed drastically except possibly at the very beginning of the moon's career, must be redone. It is concluded below that the moon melted and solidified some half-billion years after its formation. There is thus no basis for assuming that at the time of solidification i was zero. Therefore, if the value of β resulted from events during solidification and immediately afterward, we are led to conclude that, if the moon actually passed the critical point near $0.61a_0$, it did topple over on its center.

There are, then, two possibilities: (i) the moon solidified well inside the critical limit, and (ii) it solidified well outside the limit. If β is interpreted as directly due to equilibrium conditions at the time of solidification, then possibility ii is ruled out,

because the absolute value is 17 times the present equilibrium value, corresponding to the distance $(17)^{-1/3} = 0.39a_0$. Now suppose that the moon did solidify there. Its longest axis would, then, upon solidification, have pointed toward the earth. This position, however, would not have been stable in the presence of a finite orbital inclination and the small regression of the nodes as found from equations (2) and (3). This matter requires further investigation, because the results are somewhat puzzling. But a literal application of the existing theory appears to require that the longest axis would move so as to become the axis of rotation, with the intermediate axis pointing toward the earth. The third Cassini law would then take on its alternative form, by which the poles of the lunar equator and orbit would be on the *same* side of the pole of the ecliptic. This kinematical regime would last till the $0.61a_0$ limit was reached, whereupon it would change to the present regime. It is difficult to see how the initial toppling-over of the axis could have been slow enough to prevent a return to near-isostasy so shortly after solidification. This points to a general difficulty concerning the rotational stability of satellites of intermediate distance, which will require clarification before the lunar problem can be fully understood. Since the tidal bulge for the moon *outside* $0.61a_0$ would be less than one-fourth of the observed β, it is provisionally concluded that, regardless of where the moon solidified, β *cannot be interpreted as a fossil tidal bulge*. The fact that the ratio α/β deviates from the equilibrium value of $^1/_4$ is consistent with this conclusion. Stability of the present rotation merely requires $\beta > \alpha > 0$, which is, of course, actually the case.

The question remains as to whether α and β can be explained as a result of volcanism or of accretion (impacts); this has an immediate bearing on the origin of the major tectonic features on the moon. Volcanism would lead to small radial mass displacements in nearly random locations; this would not sensibly affect the *ratios* of the moments of inertia and can, therefore, not be used to explain the present values of α and β. The effects of accretion involve an analysis of the relative accretion rates on different areas, taken with respect to the moon's velocity vector; and of the rotational stability after each major accretion, with a possible reorientation of the axes. This problem will not be dealt with here; only the general remark is made that the present values of α and β are likely to give some measure of the *last* major impacts received. They suggest differential deposits in the three main directions of the order of 500 meters.

3. *Composition and Thermal History of the Moon.*—The only datum of composition known with precision is the mean density, 3.33. This is close to that of olivine (3.29), thought to be a major constituent of the earth's mantle, and probably also close to the related mean density of the outer mantle itself, 3.3 (if reduced to zero pressure). As a result, it has been rather generally held that the moon is homogeneous and composed of "mantle" material and that an iron core is absent (or exceedingly small). On the assumption of homogeneity Jeffreys[14] has computed the moment of inertia, $C = (0.3970 \pm 0.0007)MR^2$, in which M is the mass and R the radius of the moon; the slight uncertainty results from uncertainty in the amount of compression.

However, the writer finds it difficult to see how such a homogeneous object could have formed. The meteorites were derived from asteroids, bodies much smaller than the moon, and yet these bodies were molten, at least in part, and the metal

phase separated at least partially from the silicate phase. During a recent symposium on the origin of meteorites[15] the writer showed that the melting of the larger asteroids may be interpreted as due to heating of initially cool, accreted masses by their own radioactivity. With the measured ages of the *solid* earth and *solid* meteorites 4.5×10^9 and 4.6×10^9 years, respectively,[16] the probable age of the solar system may now be put very close to five billion years. At that time the heat production by U and K^{40} was about 10 times greater than now, and, for a representative mixture of meteorite material (or a more uncertain terrestrial mixture), the spheres would have heated to the melting point in less than 10^9 years, provided that the heat losses by conduction were not too large. Depending on the precise value adopted for the conductivity of loosely packed, accreted material, one finds that all spheres larger than about 100 km. in diameter will have melted at least close to the center; substantially larger spheres will have melted almost entirely, leaving only an outer shell of few kilometers of nearly unaltered accreted material. Once melting had taken place, the heat conductivity increased appreciably and convection, cooling, and freezing could set in, with the radioactive materials preferentially pressed into the surface layers, as happened on the earth. No later general melting will have occurred.

Unless the composition of the moon is very abnormal—for which there is no apparent reason—this body must have gone through the thermal cycle just described. Indeed, the lunar surface features give abundant indications that this is just what has happened (see below). Furthermore, the general sequence of events expected thermally gives a basis for the classification of the surface detail in the *premelting*, *maximum-melting*, and *postmelting* stages; this classification appears to fit the facts.

On the basis of these considerations and the fact that gravity on the moon is not negligible, it may be assumed that the moon has at least a small iron core; it need not be so large in proportion as that of the earth because the earth may have lost some of its silicates.[17] With the mean density fixed, the surface density must then be assumed to be *less* than 3.3. The coefficient in the moment of inertia must then be somewhat less than 0.40, the value for a homogeneous model, but probably well above 0.334, the value for the earth, because not only may the earth have an excess abundance of metal phase, but the compression has a large effect. There is actually one published determination of C for the moon, by de Sitter,[18] with the coefficient 0.43 ± 0.09. The large uncertainty results from the small amount of rotational momentum possessed by the moon, owing to its slow rotation, and the small exchange between rotational and orbital momentum. These factors will also hamper future determinations of C, but it is apparent that a great effort toward increased accuracy would be warranted.

It does not seem impossible, at this time, that important *direct* information on the density and constitution of the lunar surface material may become available—and may, in fact, be already at hand. The hypothesis that *tektites* originate on the moon has been advocated from time to time for decades. A recent exponent of this idea is Dr. H. H. Nininger, to whom the writer is indebted for a discussion on this subject. The specific suggestion is that impacts of meteorites on the moon cause explosions there, leading to melting of surface materials and, in some cases, to ejection from the moon, the velocity of escape being 2.4 km/sec. The hypothesis

would account for several known facts about tektites: their low age, of the order of 10^6 years or less, as determined by H. E. Suess from the A^{40}/K^{40} ratio; the fact that the bubbles occurring in the tektites have pressures not exceeding 1 mm. Hg, a value approximately required to uphold the bubbles against the surface tension of the liquid melt; the fact that they occur in showers; their very special composition, not matched or even approached by any meteorites and suggesting a fractionation process requiring a high temperature that might conceivably have occurred on the massive earth at the time of its maximum surface temperature, with some of the oxides thus lost to the earth[17] accumulated on the cold surface of the moon. If the hypothesis is correct, the density of the surface material of the moon would not exceed 2.5. This would lend support to the views on the moon's internal structure expressed before. A critical test of the tektite hypothesis will further be the study of the possible orbits which a spray of ejected material would follow upon leaving the moon—in particular, whether showers having rather *limited* dimensions on the earth can so originate. This appears necessary if the australites, the moldavites, and a few other such limited showers are to be understood. This test is currently being made by N. Metropolis and his associates with the aid of a fast electronic computer. The outcome is expected to be of great significance to the general understanding of the genesis of the earth-moon system.

4. *The Surface of the Moon.*—The surface of the moon shows such an overwhelming mass of detail, particularly under the best conditions in a large telescope, that some principles of classification are necessary to reduce this mass to comprehensible proportions. The thermal history of the moon, sketched in the preceding section, provides a basis. To begin with, one may visualize the moon having been formed quite near the earth by the settling of the condensation products (this might be called "accretion," though that term usually refers to a sweeping-up process from nebulous material not permanently associated with the body). The degree of exchange of material between the earth and the moon at that stage is unknown; but it appears probable that the two bodies originated with a common envelope, as a double planet, and not in a planet-satellite relation, from detached clouds. The accreted solid material had its radioactive heat sources spread uniformly through its mass, heated up, and began after several hundred million years to melt. The precise timing of the melting is still uncertain for two reasons: (*a*) the initial temperature of the accreted body is not known, though it follows from the observation and also from the value of M/R that it was far below the melting point, and (*b*) there is still considerable uncertainty in the abundances of the radioactive elements in a cosmic mixture of solids. The melting started near the center; but, because the thermal conductivity of the accreted material was low, most of the remaining mass melted rather soon afterward. Only a crust a few kilometers thick (the thickness depending upon the thermal conductivity) remained solid, although it was not unaltered. Large quantities of steam escaped from the interior as it heated up and passed through the crust. This led, presumably, to numerous fumaroles and to a "baking" and consolidation of most of the crust. It is well known that on the earth steam and heated solutions play a major role in changing sediments to metamorphic rocks.

The first conclusion we draw is that, while the crust consisted of loose rubble in the *premelting stage*, it consisted of hardened, metamorphic material in the *post-*

melting stage. This appears to have led to very pronounced differences in the impact craters (cf. sec. 5). The stage of *maximum melting* may be identified with the formation of the *maria*, when moderate impacts could break the crust and cause large exposures of lava. Observation suggests that the maria were not formed in one giant catastrophe. There are differences between the levels of the different maria which indicate that there are several independent formations, though the interval within which they could have formed must have been limited. For example, Mare Serenitatis has flowed into Mare Imbrium and must be presumed to be younger, while Mare Tranquilitatis has flowed toward Serenitatis and is thus younger still.

Among features that have always been puzzling are the *rills* and *ridges*. The rills are cracks in the surface, about 1″ (2 km.) in width and narrower, occurring almost anywhere on the moon (maria, old fragments of crust, crater bottoms) except in the highlands around Tycho. Often they end in a small crater, or else a small crater is placed somewhere near the center, but then the crack often makes an abrupt turn at the crater. It looks as if the impact of the crater started the crack, that the crack may have widened since, and that frequently lava has come up from below and nearly filled the crack and has given it a flat bottom. The walls of the crack are usually nearly vertical. Occasionally, as in Hyginus, the crack is lined by a string of small craters. Probability arguments show that these strings of craters cannot be the result of impacts. The Hyginus rill makes a sharp turn at Crater Hyginus, which suggests that Hyginus set off the crack. One branch of the crack points northeast. It has, first, two small craters placed squarely on the rill, separated by two or three diameters; then follow nine further craters, all in contact, after which the rill continues without any further craters. The other branch of the rill, pointing west from Hyginus, has three or four isolated small craters on it. Seen at full moon, this system shows all inside walls to be lined with a brilliant white substance, which apparently is a material having a strong back scattering, like glass beads. Presumably all these subsidiary craters are fumaroles.

The chief question is, therefore: Why did cracks and ridges form? There are two mechanisms to be examined: the change in volume accompanying the change from the solid to the liquid phase and vice versa, and thermal contraction of the solid phase, following the general freezing.

Little is known about the degree of compaction of the solid, accreted rubble before it melted or, consequently, whether or not melting of the interior caused expansion. At any rate, with the surface covered by thick layers of rubble, no visible effects may have resulted. The subsequent solidification started, presumably, from the center of the moon outward, as occurred in the mantle of the earth for the same reasons. Therefore, the first mechanism is not likely to have produced major observable effects. One must next consider the cooling of the solid moon. Clearly, the outside cooled well below the melting point first. This caused contraction of the outer shell, which rested on a hot solid interior of unchanged dimensions. Therefore, the outside shell developed tensions, and large impacts might well have set off cracks that subsequently lengthened, widened, and deepened as the cooling progressed inward and the shrinking increased. The cracks would have released pressures in the deeper layers and lowered the melting

point locally. Thus lavas might have come up through the cracks and partially filled them—although it might be difficult to predict to what extent such cracks would be filled.

Later, the cooling in the outer shell (say, 10 or 20 km.) had largely run its course, and gradually deeper and deeper layers followed. This caused the deeper layers to shrink in turn; but now the outer layers, in which the cracks had meanwhile been largely filled by new lavas, began to be compressed, since their temperatures were nearly constant but their base narrowed. *Folding* would therefore result. In this manner the ridges, visible particularly on the maria, may be explained. Also, *faults* might develop, such as indeed are known on the moon.

As is well known, the lunar surface, apart from its infinite detail, is largely of two types: *maria* and *continents*. The continental crust is supposed to be largely the original surface as built up in the process of accretion; it has been plowed and replowed by the innumerable impacts, small and large. Yet, from my visual observations I have the definite impression that there are regions which have never been disturbed by large impacts. They suggest that the moon accreted mainly from *small* particles and that the large impacts occurred at the very end—and possibly as a separate phase—of the moon's history. The maria are quite smooth, with only comparatively few craters that have rarely been damaged since their formation; in addition, many partially "drowned" craters may be seen. Especially at full moon, the maria are dark and the continents bright. But there is a *third* class of crustal rock, found especially all around Mare Imbrium and on either side of Mare Frigoris. This class consists of *lava-covered continent*. At full moon these rocks look like continent; but under oblique illumination one finds them devoid of the thousands of tiny markings that give the old accreted crust its "peppered" appearance. Instead, only smooth, coarse features are seen, and the whole area looks as if it has been covered with a thick layer of mud. The coarse features are often aligned to form broad ridges or rows of rounded mountains, which usually have a preferred direction in a given area, as if they were caused by enormous lava splashes. The *mountain ranges* surrounding Mare Imbrium—the Alps, the Caucasus, and the Apennines—belong to this third class of structure. So do many *isolated ridges* east of Crater Copernicus and near the center of the lunar disk; both groups look like jets of lava or rather lava-covered bits of old crust (or planetesimal) which have been shot from the same splashing center in Mare Imbrium. Related, but in a different category, are the many *grooves* cut in the lunar surface, particularly in high regions like old crater walls, and radiating also from Mare Imbrium. They are U-shaped in cross-section and are rarely more than five diameters long; the only exceptions I have observed are extremely shallow grooves, where the ratio may approach ten. It is natural to assume that they also have been caused by flying fragments. In some cases, like the large groove just west of Crater Herschel, the fragment is still visible at the south end of the groove. The groove is only slightly wider than the fragment. Similar, but finer, grooves surround the largest "recent" craters, like Copernicus and Tycho.

It is at first somewhat surprising that the mountain ranges around Mare Imbrium should have formed during the period of maximum melting, at which time the crust must have been fairly thin and possibly was not able to support the extra load. The *Alpine Valley* sheds interesting light on this question. It has almost

vertical walls, which have such a shape that they must have fitted together before some enormous force split them apart. It was definitely not cut by flying masses, since it is not straight or U-shaped. The valley widens toward Mare Imbrium, but its entrance to the mare is blocked by several large clumps of lava-covered rock, of the type that forms the Alps on either side of the valley. The bottom of the valley is entirely flat and smooth, clearly originally filled with liquid lava. The explanation suggested is that the crust surrounding the impact area in Mare Imbrium split open where the valley is now, that the interior lava rose to the level of the present floor, and that the entrance was filled by large *solid* blocks of crust tossed out of Mare Imbrium, covered with some lava. Since the velocity of propagation through rock was probably 7–8 km/sec, while the velocity of the ejected masses was much less than the circular velocity of 1.7 km/sec, there is no difficulty in understanding that the blocks arrived after the crust had split open. The valley proves the existence of a strong, pre-existing crust. Very probably the isolated mountains south and east of Plato are likewise solid crustal blocks tossed onto the old crust; but in this case the crust got covered also with a heavy load of liquid lava, and the crust may have broken and sunk at the inner scarp of the present Apennines. There is evidence that the crust was thinner later, when other maria formed, and that the processes were less violent than for Mare Imbrium.

5. *The Lunar Craters.*—We have seen that the moments of inertia indicate that appreciable accretions were acquired by the moon after solidification. This is consistent with the picture one obtains from the study of the lunar surface itself. If one examines the distribution of craters in a region, say, around Tycho, one is impressed by the complete *randomness* of the position of these features. Many of them are *wide, deep holes*, often flat-bottomed, with hardly any walls around them. In such instances the material was apparently blown out of these holes and scattered widely over the moon. These craters appear all to belong to the early, or premelting, era. They are inconspicuous at full moon. Tycho itself is a crater of the third, or postmelting, era. Here one sees clearly how the material was *displaced* by the impact. Prominent walls were thrown up on top of a sequence of exposed terraces very similar to Meteor Crater, Arizona, and an uncountable number of boulders were thrown to distances up to some four crater diameters. Hundreds of these boulders can be clearly distinguished in a large telescope under the best conditions, together with a very large number of cuts and grooves which they apparently inflicted on neighboring structures. But, beyond that, the entire neighborhood has a "sandy" appearance, quite contrary to regions more remote from Tycho. With proper allowance for scale, one must conclude that the number of ejected boulders runs into many thousands.

If volcanic action were responsible for the lunar craters, one would assume that a hundred or a thousand times *fewer* structures would have sufficed. Furthermore, as has been pointed out before, volcanoes would have been *additions to*, and not, as many craters are, subtractions from, the landscape. The craters form a continuous series of sizes and shapes, from the smallest visible to huge Mare Imbrium, as was stressed by Baldwin[5] (though a good relation is obtained only if structures that originated within a given era, say, the postmelting era, are intercompared). Only the impact hypothesis gives a reasonable explanation of

this family of structures. Furthermore, from the circular shape of nearly all craters, one concludes that the impacts caused *explosions*—as has been remarked by Baldwin and Urey. Volcanism, while clearly not responsible for the major tectonic processes on the moon, has, however, played an important role. Its most obvious effects are shown in the appearance of the full moon (see below), assumed to be the result of metamorphizing the lunar crust by steam and heated solutions coming from below. Further, such features as accompany Crater Hyginus (see above) could only be volcanic. Finally, important processes have occurred in crater bottoms (see below), which might be classed as volcanic.

6. *The Full Moon.*—To an observer who has gained some familiarity with the lunar features seen at the quarter-phases, the appearance of the full moon is utterly baffling. Some large craters such as Clavius are quite invisible—only some small craters inside it can be found. Other large craters can be recognized with difficulty by their ghostlike outlines. But two groups of new features have gained prominence: (1) a dozen or so large ray craters, of which Tycho is the most striking, and another dozen medium-sized ray craters; (2) thousands of small, brilliant white specks, the largest of which are either small, perfectly shaped, filled circles or white circular rings with a small dark central patch. About three days before or after full moon the features in group 2 appear to be conical craters, without appreciable rims. The brilliant, spotless white is limited with perfect precision to the inner slopes of the inverted cones. If the cones are complete (not truncated), they appear as uniformly filled circles at full moon; if they have a flat bottom, as happens in the majority of the cases, the bottoms are about as dark as the surroundings and the cones look like white rings, somewhat like the Ring Nebula in Lyra. The perfect regularity of these cones is in striking contrast with the wild and rugged appearance of most lunar structures; they look as if they were turned out on a lathe in a block of white chalk which had first been made dark gray on its surface. Mösting A, used in meridian-circle observations of the moon's position, is one of the most prominent examples of this class. The bright specks and cones occur almost anywhere, though they seem to have a preference for the equatorial belt of the moon. Those occurring in maria are often *slightly yellow* compared to the spotless white specimens on the continents, somewhat like Wood's region near Crater Aristarch.

The perfect, undamaged appearance of the white cones shows that they are among the most recent lunar structures. The same is true for the ray craters. Rays of Copernicus, Kepler, Tycho, and some others fall equally on the maria and the continents. These craters are therefore postmaria, that is, postmelting, structures. Between these two classes of "white" craters there are actually many "white" craters of intermediate size. They lack the perfect regularity of the cones but are undamaged by subsequent events. Dionysius is an example, and many more may be seen on any good photograph of the full moon. In them the white substance has usually spilled somewhat outside the bounds of the crater proper, which is also intermediate between the cones and the ray craters. It is concluded that *the white craters form one continuous class,* and the ray craters show this class to be of the *postmelting era.* By contrast, heavily damaged craters—like Clavius and nearly all other large craters in that region except Tycho—cannot or can barely be recognized at full moon; they are considered to belong to the premelting

era. The smaller white craters can be recognized under oblique illumination not merely by their undamaged outlines but also by their greater depths.

The fact that the postmelting craters are brilliant white at full moon and the premelting craters are nearly invisible demonstrates that the texture of the crust changed drastically during the melting stage. The distribution of the "white" craters over the moon further shows that the heating and melting was a *general* phenomenon and not limited to the surface layers of the present maria, as has been supposed in the recent literature. In other words, the heat causing the melting of the maria was not primarily derived from the kinetic energy of impacts but was largely pre-existing at the time of the impacts and was due to a general heating of the moon by its own radioactivity. The interval during which maria could form was thus limited. The evidence on the relative ages of lunar structures is entirely in accord with this.

A number of subsidiary problems need further investigation. What is the nature of the white substance? Is it rock flour, known from terrestrial meteor craters, or more like pulverized glass or glass beads, as suggested by the strong back-scattering power? Why are the "fumarole"-type craters along the Hyginis rill, and, in fact, the steep inside walls of this rill themselves, lined with a similar white substance? Why are the isolated mountain peaks in Mare Imbrium, south of Plato, covered with this substance and, indeed, the steep inside slopes of the Apennines?

7. *The Central Peaks.*—Among the most puzzling lunar phenomena are the central peaks found in a fraction of the craters. Some writers have regarded them as volcanoes which deposited the crater walls, thought to consist of ashes; others, as a rebound action of the soil after the impact or even as the "stuck" planetesimal itself. Consistent with the earlier parts of this paper, only the "rebound" picture needs to be considered; but, while one can visualize a rebound in a liquid or plastic, there seems no reason to suppose that a solid can act in this manner. On the earth isostatic adjustments occur only over large areas, of the order of 1,000 km. in diameter and larger. With the lower gravity on the moon, such compensation could not occur in a crater, unless the impact caused local melting or fissures which tapped a large reservoir of lava farther down. Therefore, the central peaks might consist of extrusive, igneous rock; but, if so, the question would remain as to why the extrusion occurred in some craters and not in others.

In my visual observations with the 82-inch telescope I have paid much attention to the central peaks, and my general impression is that they are igneous masses, pressed up as lavas through the crater bottoms. They do not look like rubble or fractured material, as the crater walls do. Instead, they rise as majestic peaks, singly or several in a group, like simple, "functional" structures; often they have sharp, sloping ridges and crests, giving the mountain an angular appearance. Often, besides impressive central mountains, one finds groups of lower, rounded mountains over a good part of the crater floor or surrounding the central peaks; they may show the flow lines of squeezed-out lava. Crater Franklin is a good example. Only very exceptionally does one find a central mountain mass that looks very different from this; the strangest case is Crater Alpetragius, which at certain illuminations resembles a bird's nest containing two eggs, one large and one small. One might wonder whether in this case one actually sees the plane-

tesimal. (One does occasionally find *grooves* where one can see the fragment that apparently caused it, stuck at the end of the groove.)

If the central peaks were squeezed up as lavas, this would seem to imply two things: (*a*) there was previously a lack of isostatic compensation or at least an excess pressure under the new crater, and (*b*) a lava reservoir was available. The first conclusion is consistent with a striking group of three craters near the center of the lunar disk, Ptolemy, Alphonsus, and Arzachel, which seem not very different in age; all three show damage done by flying fragments from Mare Imbrium, and, from the lack of serious damage by other impacts, they may be classed as late premelting era. Observation shows that the bottom levels of these craters are very different, decreasing in level in the order in which the craters were mentioned. The central peaks vary in the opposite sense. Ptolemy has none, Alphonsus a small one, and Arzachel a huge one.

The second condition implies that craters formed in the early premelting era probably could not form central peaks, while those formed shortly before, during, and shortly after the formation of the maria could; and, again, very late postmare craters could not. This accords well with the observations. The many very early formations around Tycho and near the south pole, referred to before, have no central peaks. Conversely, craters with central peaks are, on the whole, fairly well preserved and cannot therefore, as a group, be early premelting. It is not unlikely that in marginal cases the heat of impact contributed to the availability of a lava reservoir; but the relations stated on the presence and absence of peaks indicate that the impacts were not the principal cause of the lava.

Related to this question of isostatic compensation are the craters whose bottoms have been clearly raised since their formation, some to the point of overflow. Of the latter group, Wargentin is the famous example; another case, near the center of the disk, is a medium-sized crater just south of Godin. In both cases a thin old crust was apparently lifted by the upwelling lava, although a few patches of dark lava show through; this is seen at full moon. There are very many other spots on the moon where the lava apparently welled up through the crust and is now causing small and large dark patches all over the moon, except near Tycho and the south pole, where the crust has held together. A very remarkable event occurred in Crater Posidonius, where the upwelling lava apparently lifted the entire crater bottom and rammed it at a higher level sideways against the southwest crater wall, causing it to break in two and causing four parallel ridges of folded mountains to form on the old floor. The break occurred not precisely at the rim of the floor but somewhat farther out, so that the lower terrace of the wall was included. It was lifted to the level of the outer terrace. The opposite side left the lava exposed, which shows as a darker crescent around the old floor. (The unusual side motion may have occurred when Mare Imbrium formed.) These instances are but a few that could be cited, showing very clearly that the lava came from *below* and not from one or two impact areas, like Mare Imbrium. It resulted in many isolated small and larger patches of lava all over the surface of the moon. Flow from one or two central hearths would have caused systematic level differences between the lava fields, for which there is no evidence.

8. *The Origin of the Planetesimals.*—The origin of the bodies that struck the moon and formed the craters and maria may be thought to be one of these four

possibilities: (1) they formed inside "protomoon," or rather from condensations in the inner part of the earth-moon cloud; (2) they formed inside protoearth, but outside the original moon's orbit; this would imply that this part of protoearth was too tenuous to become gravitationally unstable and form a second, outer satellite around the earth; (3) they formed between Mars and the earth in a part of the solar nebula claimed by neither planet, with the bodies initially moving around the sun and, as a result of perturbations, having gradually been swept up by collisions with planets, somewhat along the lines considered by Öpik;[19] and (4) they formed in the asteroid belt, coming to the moon as meteorites or small asteroids.

At present, process 4 is operating both on the earth and on the moon. Because of the increasingly rapid rate of pulverization of the asteroid ring,[20] the present rate of process 4 is expected to be much greater than the rate during the earliest period of the planetary system. Since the moon shows an enormous impact rate a few hundred million years after its formation, and only a very low rate since, as is shown by the maria, process 4 is definitely ruled out for this early period. Process 3 is entirely speculative, and there seems no reason why it should be peaked in time or have the enormous intensity as required. Process 1 must be responsible for the bulk of the moon's mass, but it seems inexplicable why it should have had a renewed burst of activity several hundred million years after its main activity. This leaves process 2 as the most likely. As has been pointed out elsewhere,[21] one feature of the sequence of lunar impacts is accounted for by process 2: the fact that some of the largest impacts came toward the end of the bombardment. It is explained as due to the slow sedimentation rate toward the planet of large planetesimals.

One may make a further check on the nature of the impacts from energy considerations. In section 2 it was found that the inequalities of the surface deposits are of the order of 500 meters if averaged over areas roughly the size of octants. They could be much larger locally. Now the energy of impact per unit area is $^1/_2mv^2$. Take an area 100 km. square. The deposited mass will be of the order of $10^{14} \times 5.10^4 \times 3 = 1.5 \times 10^{19}$ gm., if the density of the material is 3. Baldwin[22] has given the empirical relation between crater diameter and energy for terrestrial explosion pits. The extrapolation to the very large lunar craters is quite uncertain because the plot by Baldwin shows some curvature. Baldwin's relation would give 10^{15} cal. for $d = 3$ km., $10^{18\,1/2}$ cal. for $d = 30$ km., and 10^{22} cal. for $d = 300$ km. These figures may be checked by the following computation: The mass of the displaced material is proportional to d^2 times the crater depth. The latter increases as $d^{2/3}$ for medium- and large-sized craters, according to another compilation by Baldwin.[5] The mass increases, therefore, as $d^{8/3}$. The range of the displaced material increases as d, and therefore the energy per unit mass also. Accordingly, the energy needed to make a crater should increase as $d^{11/3}$, which supports the $d^{7/2}$ relation based on Baldwin's extrapolation. For a crater 30 km. in diameter one may thus estimate $E = 10^{19}$ cal. and use the 11/3 power law beyond.

The number of lunar craters per 100 km. square has been roughly estimated for groups of d increasing by factors $\sqrt{10}$. The results are 3 craters of $d = 30$ km., 0.1 of $d = 100$ km., 0.005 of $d = 300$ km., and 0.0005 of $d = 1{,}000$ km. The last item represents one Mare Imbrium on a hemisphere. The corresponding numbers per octant would be 1,500, 50, $2^1/_2$, and $^1/_4$, respectively. Clearly, the first two

groups are so numerous that random fluctuations will be insufficient to cause appreciable effects on the differences of the moments of inertia. We need to consider only the two groups of maria, with $d = 300$ and 1,000 km. They give 10^{28} and 8×10^{28} ergs, respectively, per square of 100 km. The energy equation now gives $v = 1.1$ km/sec as the velocity of the incoming bodies. In fact, only the Imbrium impact really matters; the remaining contributions are merely in the nature of minor corrections.

The data used are admittedly rough and somewhat hypothetical, partly because of the unknown contribution to α and β by the invisible side of the moon and partly because it is not known how much α and β have decreased since the impacts. The latter uncertainty makes v(computed) an upper limit; the former might introduce an uncertainty of about $\sqrt{2}$. The energies might be uncertain by a factor approaching 10 and because of this the velocity by a factor approaching 3. All considered, it is very probable that the impacts occurred with velocities not exceeding the velocity of escape, 2.4 km/sec. This rules out processes 3 and 4 listed above. Further, one concludes that the axis pointed toward the earth is the one on the extremities of which the largest impacts occurred. Mare Imbrium is foremost among these. One should not assume that the moon had the present orientation with respect to the earth before these maria formed. More probably the side of Mare Imbrium was either trailing or advancing in the orbit.

If we adopt the impact velocity as 2.4 km/sec, the velocity of escape, the masses and dimensions of the planetesimals may be computed. The Imbrium planetesimal comes out to be about 6×10^{21} gm. or some 150 km. in diameter. The next group of 10 bodies that caused the $d = 300$ km. group would have averaged about 7×10^{19} gm. and $d = 35$ km. Following this group would be the group of 200 bodies averaging 10^{18} gm. and $d = 2$ km. Actually, these numbers were found for one hemisphere only and should therefore be doubled. The total mass contributed to the moon by the four groups (with the factor 2 applied) is, then, from the largest bodies downward, 1.1×10^{22}, 1.3×10^{21}, 5×10^{20}, and 1.7×10^{20} gm., respectively. If the figures used are at all representative, they indicate that the mass contribution has been made largely by the few large impacts, in spite of the thousands of smaller bodies collected. Since the mass of the moon is now 7×10^{25} gm., the total addition during the impact period appears to have been roughly one-five-thousandth of the moon's mass. A similar addition would have been estimated directly from the moments of inertia had it been known beforehand that very few additions are responsible for the inequalities.

It is found, then, that the moon, on its outward journey from the earth resulting from tidal friction, struck a region in space where it was bombarded with a swarm of objects that, from the small impact velocities, must be considered to have been moving around the earth in or near the plane of the ecliptic. Such a ring of bodies has been called a *sediment ring*.[21] The asteroid ring and the ring of Saturn are the two cases at present directly observable. Sediment rings will form where the density of the nebula is insufficient to cause breakup by gravitational instability.[21] This sediment ring must have formed well outside the original moon's orbit, in a region of protoearth where the density was well below the critical limit referred to. The small total mass swept up by the moon is consistent with this concept.

The *dimensions* to which the minor satellites in the sediment ring could grow may now be estimated. Suppose we consider the region to have been near $a = 0.4a_0$. The Roche density in the tidal field of the earth is there 0.003 c.g.s. The fact that no major satellite formed puts the density below the critical limit, which is found to be 0.0003 c.g.s. if one adopts 100° K. for the temperature (in the earlier stages of protoearth the sun had not yet completed its contraction) and, further, 2.4 for the mean molecular weight, the value applicable to cosmic matter. If one takes the actual density to be 0.1 of the upper limit stated, one may compute the accretion rate. Using $\bar{\mu} = 50$ for the condensable molecules and 0.005 as their total mass fraction in the cloud,[21] one finds the accretion rate in the diameter of the planetesimals to be 200 meters per year if *all* condensable molecules stick. This is the upper limit; besides, the actual density of the cloud may have been less than supposed, and the ring may have been out somewhat farther than $0.4a_0$. Nonetheless, it is clear that there is no difficulty in assuming that the sediment ring produced bodies up to 10 or even 100 km. in size. The absence of such bodies would be more puzzling than their presence. These bodies, however, being much smaller than the moon, could not have been first molten and then solidified before they struck the moon during its melting stage. They would probably have been heated enough to have become partially consolidated by the action of water and steam. Only the Imbrium satellite might have approached the melting point in its interior.

The surface of the moon offers a unique opportunity to study the composition of a sediment ring some half-billion years after its formation. It is regarded as a coincidence that the moon struck this ring near the time of its maximum melting. It seems not unlikely that further studies could define with some precision where the sediment ring occurred and what this implies for the place of birth of the moon.

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*THE ROLE OF NUTRITION IN THE HOST-PARASITE RELATIONSHIP**

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The concept of virulence is intimately associated with the nature of the host's response to an invasion by a parasite. The successful establishment, with attendant reactions, of a parasite in a host serves as the basis for considering the parasite to be virulent and, at the same time, the host to be susceptible. On the other hand, failure on the part of the parasite to establish itself in a host may be either a reflection of the avirulence of the parasite or a manifestation of the resistance of the host. Consequently, virulence may be defined as an expression of a host-parasite relationship. Such a definition is essentially ecological, emphasizing the importance of the host as an environment for the parasite. The property of virulence with respect to the parasite assumes that the parasite is capable of utilizing the host environment as a growth medium and that the parasite can overcome the host's defense mechanisms. Although the role of the nutritional status of the host with respect to its resistance or susceptibility to a specific pathogen has not been minimized, the relationship between this factor and the specific nutritional requirements of the pathogen has been subjected only comparatively recently to direct investigation.[1–6]

This report presents preliminary data on the role of nutrition in a host-parasite relationship involving a system not complicated by the intricate defense mechanisms of the animal host, i.e., a pathogen attacking plant tissue.

Materials and Methods.—A bacterial plant pathogen, *Erwinia aroideae*, strain RK,[4] was selected for several reasons. The A medium was satisfactory whenever a synthetic medium was required;[7] Noble agar (Difco) was added to a concentration of 1.5 per cent to prepare solid media. Biochemical mutants were obtained by means of ultraviolet irradiation, followed by penicillin treatment.[8] The mutant yield, however, was relatively low, approximately one mutant per 10^3 tested colonies. The nutritional requirements of these mutants were determined auxanographically[9] and were apparently specific in that no other commercially available amino acid, purine, pyrimidine, vitamin, or growth factor would support growth, with a single exception: citrulline supported the growth of the mutant requiring arginine. This species is a comparatively omnivorous pathogen attacking several host species. Of the five host species used, the carrot was decidedly inferior to the potato, cucumber, radish, or turnip with respect to the clarity of host response

to inoculation. Finally, washed suspensions (*ca.* 10^8–10^9 viable cells) of the parental strain and virulent mutants added to the cut surface of these hosts produced a distinctive discoloration within 24 hours when incubated in a humid chamber at *ca.* 27° C., and eventually rot. All tests of the virulence of the biochemical mutants were repeated at least twice.

Seven biochemical mutants, each requiring either one or two amino acids, were available. Biochemical mutants of *Klebsiella pneumoniae* with comparable requirements were used in certain studies.

Blocks and homogenates of tissues were added auxanographically to pour plates of the mutants of *E. aroideae* and of *K. pneumoniae* in solid synthetic medium to determine whether these supplements would support growth. The specific amino acid was also added as a control.

Results.—Cut surfaces of the cucumber, potato, radish, turnip, and carrot were inoculated with thoroughly washed suspensions of biochemical mutants of *E. aroideae* requiring the following: methionine, arginine, isoleucine-valine, threonine, leucine, cysteine, and histidine. The host response for each mutant is summarized, in Table 1. All mutants were virulent for the cucumber; only the mutant requiring

TABLE 1

THE VIRULENCE OF BIOCHEMICAL MUTANTS OF *Erwinia aroideae* AND THEIR RESPONSE TO BLOCKS AND HOMOGENATES OF TISSUE OF FIVE HOST SPECIES

	Host Species*													
	Cucumber		Potato			Radish			Turnip			Carrot		
Mutant Need	Vir	Bl	Vir	Bl	Hom	Vir	Bl	Hom	Vir	Bl	Hom	Vir	Bl	Hom
Methionine	+	+	+	+	−	+	+	−	+	+	+	+,0	+,0	+
Arginine	+	+	+	+	−	0	0	+	0	0	+	+,0	+,0	+
Isoleucine-valine	+	+	+	+	−	+	+	−	0	0	+	0	0	+
Threonine	+	+	+	+	−	0	0	+	0	0	+	0	0	+
Leucine	+	+	+	+	−	+	+	−	+,0	+,0	+	+	+	−
Cysteine	+	+	+,0	+,0	+	0	0	+	+	+	+	0	0	+
Histidine	+	+	+	+	−	+	+	−	+	+	+	0	0	+

* Vir = virulence on cut surface; Bl = growth of bacteria on plates supplemented auxanographically with tissue blocks; Hom = growth of bacteria on plates supplemented auxanographically with tissue homogenates.

cysteine displayed either low or no virulence for the potato. Three types of reactions were noted for the turnip and carrot: always virulent, always avirulent, and a variable response of being virulent to some degree or avirulent. It should be mentioned in this connection that the vegetables were purchased in markets over a period of time, and it is possible that different varieties or a variety grown under different conditions were used. The virulence of biochemical mutants for different varieties of a host species is a problem currently under investigation. In the case of the turnip, mutants requiring arginine, isoleucine-valine, or threonine were always avirulent; mutants requiring methionine, cysteine, or histidine were always virulent; and the mutant requiring leucine was variable. For the carrot, mutants requiring isoleucine-valine, threonine, cysteine, or histidine were always avirulent; the mutant requiring leucine was always virulent; and the mutants requiring methionine or arginine were variable. For the radish, mutants requiring arginine or cysteine were always avirulent; the mutants requiring methionine, isoleucine-valine, leucine, or histidine were always virulent.

Since a preliminary study had indicated that the virulence of a biochemical mutant of *E. aroideae* for certain host species was correlated with the ability of these

hosts to satisfy the nutritional requirement of the mutant, blocks of tissues of each of the five host species mentioned were added as auxanographic supplements for each of the seven biochemical mutants. In each case, virulence was correlated with a growth response and avirulence with no growth response. Mutants displaying a variable virulence also displayed a variable growth response, i.e., when a virulent reaction was observed, the tissue block supplied the required nutrilite, and when there was an avirulent reaction, the tissue block did not supply the required nutrilite. These results are summarized in Table 1.

Tissue homogenates were added as auxanographic supplements for those mutants which were avirulent for certain host species to determine whether the tissue contained the required nutrilite which was apparently absent on the basis of a lack of a growth response when a tissue block was added as a supplement. In every case the tissue homogenate supplied the required nutrilite (Table 1). This observation suggested that the required nutrilite may be present in a tissue but not be available either at the cut surface or when supplied by a tissue block.

Assuming that the cut surface or a block of tissue did contain required nutrilites as indicated by positive responses for virulent mutants, it was considered possible that the concentration of certain amino acids may have been too low or that certain biochemical mutants of *E. aroideae* may have required a greater concentration than was available. Since preliminary evidence had indicated that an arginine-requiring mutant of *E. aroideae* and of *Escherichia coli* displayed a similar response to tissue blocks, biochemical mutants of *K. pneumoniae* were next considered.[3] The results of adding tissue blocks of the potato, radish, turnip, and carrot to mutants of *K. pneumoniae* with nutritional requirements comparable to those of the mutants of *E. aroideae* are summarized in Table 2. Tissue blocks of the potato and

TABLE 2

THE RESPONSE OF BIOCHEMICAL OF MUTANTS OF *Klebsiella pneumoniae* TO BLOCKS AND HOMOGENATES OF TISSUE OF THE POTATO, RADISH, TURNIP, AND CARROT

	Host Species*							
	Potato		Radish		Turnip		Carrot	
Mutant Need	Bl	Hom	Bl	Hom	Bl	Hom	Bl	Hom
Methionine	+	−	+	−	+	+	+	+
Arginine	+	−	+	+	+	+	+, 0	+
Isoleucine-valine	+	−	+	−	+	+	+, 0	+
Threonine	+	−	+	−	+	+	+	+
Leucine	+	−	+	+	+, 0	+	+	+
Cysteine	+	−	+	+	+	+	+	+
Histidine	+	−	+	−	+	+	+	+

* Bl = tissue block; Hom = tissue homogenate.

radish added to mutants of *K. pneumoniae* requiring arginine, threonine, or cysteine gave a positive growth response, indicating that a sufficient amount of these amino acids was present to support the growth of these biochemical mutants. The turnip blocks gave a positive response for the mutants requiring arginine, isoleucine-valine, or threonine but a variable response for the mutant requiring leucine. Carrot blocks yielded a positive response for the mutants requiring methionine, threonine, cysteine, or leucine but a variable response for the mutants requiring arginine or isoleucine-valine. In all cases in which the tissue blocks did not always supply the needed nutrilites, tissue homogenates gave positive results (Table 2) These results indicated that in many instances the cut surfaces or blocks of tissue

did contain the required nutrilites for certain mutants of *E. aroideae*, but the comparable mutants of *K. pneumoniae* may have required smaller concentrations for growth.

Experiments were designed to test comparable mutants of *E. aroideae* and *K. pneumoniae* for their ability to grow in a series of concentrations of filtrates of radish homogenate as well as the specific amino acid (Tables 3 and 4). Biochemical mutants requiring the following were tested: methionine, threonine, cysteine, and arginine. Both the filtrates and the solutions of amino acids were added as auxanographic supplements. The experiments involving the amino acids were conducted by W. R. Lockhart, of the Department of Bacteriology, Iowa State College. All four biochemical mutants of *K. pneumoniae* gave better growth responses than the comparable mutants of *E. aroideae* for both types of supplementation.

TABLE 3

THE RESPONSE OF BIOCHEMICAL MUTANTS OF *Erwinia aroideae* (Ea) AND *Klebsiella pneumoniae* (Kp) TO A SERIES OF DILUTIONS OF A FILTRATE OF RADISH HOMOGENATE ADDED AUXANOGRAPHICALLY*

FILTRATE CONCENTRATION	NUTRITIONAL REQUIREMENTS OF MUTANTS							
	Methionine		Threonine		Cysteine		Arginine	
	Ea	Kp	Ea	Kp	Ea	Kp	Ea	Kp
1.0	2+	3+	1+	3+	2+	3+	1+	3+
0.1	0	0	0	2+	0	1+	0	0
0.05	0	0	0	1+	0	0	0	0

* 0.05 ml. of filtrate added to a sterile filter-paper pad placed on surface of pour plate of mutant in minimal medium. 3+ = excellent; 2+ = good; 1+ = poor; 0 = dubious or none.

TABLE 4

THE RESPONSE OF BIOCHEMICAL MUTANTS OF *Erwinia aroideae* (Ea) AND *Klebsiella pneumoniae* (Kp) TO A SERIES OF CONCENTRATIONS OF AMINO ACIDS ADDED AUXANOGRAPHICALLY*

AMOUNT OF AMINO ACID (μGM.)	NUTRITIONAL REQUIREMENTS OF MUTANTS							
	Methionine		Threonine		Cysteine		Arginine	
	Ea	Kp	Ea	Kp	Ea	Kp	Ea	Kp
100	2+	3+	2+	3+	2+	3+	2+	2+
5	1+	3+	0	2+	0	2+	2+	2+
1	0	0	0	2+	0	0	0	1+
0.5	0	0	0	2+	0	0	0	0

* 0.05 ml. of solution containing the absolute amount of amino acid added to a sterile filter-paper pad placed on surface of pour plate of mutant in minimal medium. Amino acids: *dl*-methionine; *dl*-threonine; *l*-cysteine·HCl; *l*-arginine·HCl. 3+ = excellent; 2+ = good; 1+ = poor; 0 = dubious or none.

Discussion.—The virulence of specific biochemical mutants of *E. aroideae* for certain host species was related to the availability of the required nutrilites at the cut surface of the tissue; the avirulence of specific biochemical mutants for certain host species was also related to the unavailability of the required nutrilites at the same site. That a tissue contained the nutrilites required by the avirulent mutants was indicated by the growth of these mutants when tissue homogenates were added as supplement. Moreover, in several cases tissue blocks were found to contain a sufficient amount of required nutrilites to support the growth of mutants of *K. pneumoniae* with nutritional requirements comparable to those mutants of *E. aroideae* which were avirulent for the hosts supplying the tissue blocks. These observations have suggested a probable interpretation for the avirulence of specific biochemical mutants for certain host species.

The role of nutrition in the host-parasite relationship may be referred to the nutritional requirements of the parasite as they may be satisfied by the host. Spe-

cifically, the parasite must obtain all its nutritional requirements from the host at the site either of entry or of localization. It also appears reasonable to assume that the mere presence of the required nutrilites as detected by such physicochemical methods as paper-partition chromatography need not indicate that they will be available to parasites requiring these nutrilites. The ability of a specific parasite to utilize the available nutrilites at the site of entry or localization may determine whether or not a population can proliferate or metabolize. Although it is possible that the avirulence of specific biochemical mutants for certain host species may indicate the presence of "inhibitors" in these hosts, this interpretation does not seem to explain all the observations noted in this investigation as well as does the proposed interpretation.

Formal *et al.*[10] have recently presented a study of a strain of *Salmonella typhosa* avirulent for the mouse, indicating that a spontaneous mutation to a requirement for purine was responsible for the loss of virulence. Seeler and Ott[11] and Brackett *et al.*[12] have demonstrated an increased resistance or, stated differently, a diminished virulence for avian malaria in chicks which were either riboflavin- or pantothenic acid–deficient, respectively. These observations suggest that the avian malaria parasites may require these growth substances and probably were unable to find a sufficient concentration in the deficient host either to proliferate or to metabolize.

The avirulence of certain biochemical mutants of animal pathogens has been investigated, using the mouse as host and the following bacterial species: *S. typhosa*,[1, 2] *K. pneumoniae*,[3] *Salmonella typhimurium*,[5] and *Pasteurella pestis*.[13] With the exception of *S. typhimurium*, all mutants requiring purine had either diminished or no virulence. In these species the loss of virulence was directly correlated with the unavailability of purine in the host. In *S. typhimurium*, however, several mutants requiring purine did not display a loss of virulence. None of the tests accomplished with the other bacterial species mentioned, to determine whether or not purine availability was responsible for the loss of virulence, was done with the purine-requiring mutants of *S. typhimurium*. It would be interesting to determine why certain mutants of this pathogen requiring purine did not conform. Unfortunately, this type of investigation on the virulence of biochemical mutants of animal pathogens has been restricted to a single host species, the mouse.

Keitt and Boone[6] have presented data on the virulence of biochemical mutants of the fungus *Venturia inaequalis*, the causative agent of apple scab. A considerable number of mutants requiring amino acids, purines, pyrimidines, vitamins, or growth factors was tested. As in the case of *S. typhimurium*, several mutants requiring purine had lost their virulence, while others had only diminished virulence. A similar situation was noted for mutants requiring pyrimidine, histidine, or methionine. No interpretation was proposed for the different degrees of virulence for mutants with the same nutritional requirements, although it was established that avirulence was correlated with auxotrophy. It is possible that biochemical mutants with similar growth requirements may differ in their ability to utilize whatever concentration of the required nutrilite may be present in the host at the site either of entry or of localization.

Assuming that the virulence of pathogens with specific nutritional requirements may involve the presence of these required nutrilites at utilizable concentrations in

the host, it may be fruitful to speculate on the role of nutrition in the host-parasite relationship. It should be emphasized that, if a parasite cannot proliferate or metabolize in a host, it cannot be virulent. The reverse situation does not automatically confer virulence, since many factors are responsible for virulence or, in the case of the host, resistance. The specificity of obligate parasites for certain host species may in part be a matter of the satisfying of the nutritional requirements of the pathogen by the host environment. If differences in the virulence of biochemical mutants with similar nutritional requirements reflect differences in their ability to proliferate or to metabolize in a given amount of required nutrilites, this interpretation may apply to differences in the virulence of strains of a species of obligate parasite or pathogen with exacting nutritional requirements. The resistance of the host to specific pathogens is often affected by the host's nutritional status, which may, in turn, affect the concentration of required nutrilites at the site of entry or localization. Bacon *et al.*[2] have demonstrated such a relationship between the virulence of a *p*-aminobenzoic acid–requiring mutant of *S. typhosa and the* amount of this nutrilite supplied in the diet of the host (mouse). The resistance of varieties of plants is often genetic in origin, but the mechanism whereby the genotype accomplishes this phenotype is generally not entirely clear. It is possible that not only may materials be elaborated which provide a hostile environment for the parasite but also the concentrations of nutrilites required by the parasite may be lowered to a level beyond utilization by a proliferating or metabolizing population.

Summary.—Seven biochemical mutants of *E. aroideae*, a bacterial plant pathogen, each requiring one or two amino acids, were tested for their virulence on the following host species: cucumber, potato, turnip, radish, and carrot. Specific mutants were avirulent for certain host species. Virulent mutants gave a positive growth response when blocks of tissue from the attacked hosts were added as auxanographic supplements; avirulent mutants yielded no response when blocks of tissues from resistant hosts were added as auxanographic supplements. Tissue homogenates from resistant hosts provided the required nutrilites when added as auxanographic supplements to the avirulent mutants. Biochemical mutants of *K. pneumoniae* with nutritional requirements comparable to those of the avirulent mutants of *E. aroideae* gave a positive growth response when tissue blocks from resistant hosts were added as auxanographic supplements. Four biochemical mutants each of *E. aroideae* and *K. pneumoniae* with comparable requirements were tested for their relative response to a series of concentrations of radish homogenate and solutions of the specific amino acid. The mutants *K. pneumoniae* responded better than comparable mutants of *E. aroideae* to both types of supplementation. The avirulence of specific biochemical mutants for certain host species was related to the inability of these mutants to utilize the available concentration of the required nutrilite at the cut surface of the host. The role of nutrition in the host-parasite relationship was discussed.

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*THE RELATION BETWEEN MODULATOR AND ACTIVATOR IN MAIZE**

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Variegated pericarp, which Emerson[1] first interpreted in terms of a somatically unstable gene, P^{vv}, is a widely occurring character in maize, presumably of great antiquity. Brink and Nilan[2] postulated that the variegated allele is a compound structure comprising the P^{RR} gene (self-red pericarp and cob) and an accessory element termed "Modulator" (*Mp*) which is separable from the *P* locus. It is now shown that the unit which is auxiliary to P^{RR} in variegated pericarp is similar to, if not identical with, Activator (*Ac*), which McClintock[3] has established as the co-agent of Dissociation (*Ds*) in causing instability and chromosome breakage at other loci at which the latter element is located.

McClintock identified Activator in a stock not carrying variegated pericarp in which various structural changes in the chromosomes had occurred as a result of the operation of the chromosomal type of breakage-fusion-bridge cycle in early sporophytic mitoses. Modulator, as mentioned above, was first recognized in quite different cultures as a component of the P^{vv} allele. Although McClintock observed that *Ac* ordinarily gave a detectable effect only in conjunction with *Ds*, located elsewhere in the genome, whereas *Mp* was unitary and autonomous, the early evidence showed that Activator and Modulator resembled each other in two respects. In the first place, the dosage effects, even though measured by different criteria in unrelated strains, were parallel (Brink[4]). Second, both *Ac* and *Mp* were capable of shifting from one position to another in the chromosome complement. Critical tests of the correspondence between the two elements were needed. One such test has now been made by determining that Modulator, like Activator, in the presence of Dissociation, regularly incites chromosome breakage at the position of the latter unit.

The Pericarp Strains Tested.—It was of interest to determine whether, among the diverse array of alleles at the *P* locus, the capacity to promote *Ds* chromosome breakage was limited to variegated. The *P* locus on the short arm of the first chromosome is one of the most complex in maize. Four classes of stable alleles commonly occur, as well as a mutable type, mosaic, distinct from variegated.

Mosaic pericarp appears in a wider variety of patterns than variegated; and, whereas the latter mutates regularly to stable self-red, it is characteristic of mosaic to change very frequently from one unstable form to another. These alleles, together with two previously unreported types, designated as "Boyaca" and "Q36," are listed in Table 1. They were derived from widely different geographic

TABLE 1

TESTS OF DIFFERENT *P* ALLELES TO PROMOTE *Ds* CHROMOSOME BREAKS

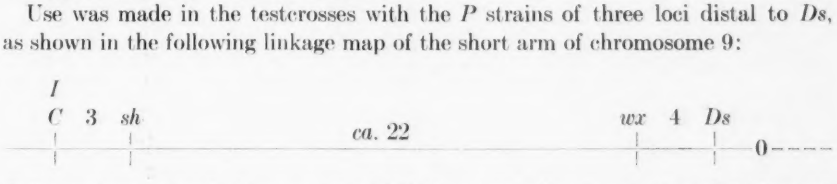
CLASS OF *P* ALLELE	PHENOTYPE		NO. STRAINS TESTED	GEOG. SOURCE OF ALLELES	RESULTS BY STRAINS	
	Pericarp	Cob			Positive	Negative
		Stable Alleles				
PRR, *POR*, *POO*	Red or orange	Red or orange	9	N. Amer.; S. Amer.	0	9
PWR	Colorless	Red	8	U.S.	0	8
PCW	Reddish	Colorless	3	U.S.	0	3
PWW	Colorless	Colorless	6	U.S.	0	6
Boyaca	Finely striped	Reddish	1	Colombia	0	1
		Unstable Alleles				
PVV	"Variegated"	"Variegated"	8	N. Amer.; S. Amer.	8	0
PMM	"Mosaic"	"Mosaic"	5	N. Amer.; S. Amer.	0	5
Q36	"Striped"	Red	1	Quebec	0	1

sources, varied appreciably in phenotype within classes, and probably represent a fair sample of the existing diversity at the *P* locus.

Tester Stocks for a Promoter of Dissociation Events (Ac-Testers).—Chromosome breakage arising directly from *Ds* activity is conveniently detected by the use of marker characters expressed in the endosperm. Breakage at *Ds* in a chromosome carrying distally located dominant genes, followed by loss to the nucleus of the acentric fragment, is disclosed by the appearance of an area of different phenotype if the other two chromosomes in the triploid endosperm set possess the corresponding recessives. Size of a mutant area is directly related to the stage in endosperm development at which the *Ds* break occurs. McClintock[3] found that the timing of *Ds* events is controlled by the "state" (mutant form) and dosage of Activator.

McClintock furnished us with stocks carrying *Ds* in its "standard" position between the waxy locus and the centromere in the short arm of chromosome 9. These strains lacked *Ac*. *Ds* events could occur in them, therefore, only if *Ac*, or an *Ac*-like element, were introduced. Our purpose was to determine whether the latent *Ds* present was "activated" on being brought into association with variegated pericarp or other *P* alleles.

Use was made in the testcrosses with the *P* strains of three loci distal to *Ds*, as shown in the following linkage map of the short arm of chromosome 9:



C determines colored aleurone, provided that two other genes, *A* and *R*, located in different chromosomes, are present also. The recessive gene *c* gives a colorless phenotype in the combination *A R c*. *I*, an allele of *C* and *c*, is a dominant inhibitor of color, so that the aleurone in normal *A R I/C* kernels is colorless. Provided that the endosperm is *R R R* or *R R r*, the contrasting effects of *C* and *c* and of *I* and *C* are detectable even in small patches. The *sh* (shrunken) gene

results in a hollow endosperm, whereas a normal (*Sh*) endosperm is more nearly solid. The carbohydrate reserves in *wx* (waxy) endosperm stain reddish-brown with iodine, in contrast to the purplish-black reaction of ordinary starch formed under the action of *Wx*. Waxy areas, both large and small, are readily identified in kernels which otherwise are nonwaxy.

Detection of Ds Chromosome Breakage.—Two procedures were used in scoring the *P* alleles for capacity to promote *Ds* chromosome breakage. These will be referred to as the *direct* and *two-step* methods. Both avoided the difficulties of classification arising from *R*-spotting. (The *R* gene is complementary to *C* in producing color in the triploid aleurone cells. In the presence of *C*, *R R R* and *R R r* give self-color, but the *R r r* genotype sometimes results in a type of spotting simulating the mottled pattern given by *C* to *c* phenotypic changes arising from *Ds* action in *C Ds/c ds/c ds* tissue.) Most of the *P* stocks tested were *c c*, a few were *C C*, and all were *r r*.

The direct method involved placing pollen from an *r r c c* individual carrying a particular *P* allele on the silks of a tester plant homozygous *R*, *C*, and *Ds* (standard position), but lacking *Ac*. Successive or concurrent breaks at *Ds* in the two chromosomes of maternal origin in the resulting *C Ds/C Ds/c ds* kernels give rise to a lineage of colorless cells as a result of loss in the acentric fragments of both *C* genes.

The two-step procedure was used when the *P*-stock × *Ac*-tester cross was of the *r r* ♀ × *R R* ♂ type. Following such matings, F_1 kernels which appeared to be showing *C* to *c* (colored to colorless) or *I* to *C* (colorless to colored) mottling, superposed on the *R*-spotting present, were selected for propagation. The resulting F_1 plants were selfed, and pollen from them was placed on *c sh wx*, no-*Ac*, *R* individuals. This testcross yielded kernels, free of *R*-spotting, which could be examined for *Ds* breakage events expressed as losses of the dominant marker characters at the *c*, *sh*, and *wx* loci.

The P^{VV} Allele as a Promoter of Ds Chromosome Breakage.—The results of the tests of the different *P* alleles to promote *Ds* chromosome breakage are summarized in Table 1. The four commonly occurring classes of stable genes, P^{RR}, P^{WR}, P^{CW}, and P^{WW}, gave negative results throughout. The same applies to the stable Boyaca allele. Likewise, the five representatives tested of the highly mutable allele, mosaic pericarp, did not promote *Ds* chromosome breakage. The unstable *P* allele carried by the Q36 stock also gave negative results. In contrast, all the P^{VV} alleles, from eight different geographic sources, regularly gave rise to *Ds* mottling. It is evident that variegated alone among the diverse array of *P* alleles, stable and unstable, has the capacity to promote *Ds* activity.

The different *P* alleles giving color in the pericarp were tested for stimulation of *Ds* activity in plants heterozygous with P^{WR} or P^{WW}. These latter genes were derived from known inbred lines of dent corn, which, on being tested for capacity to promote *Ds* chromosome breakage, were found to be negative. One may conclude, therefore, that in the P^{VV}/P^{WR} and P^{VV}/P^{WW} heterozygotes scored it was the P^{VV} allele present that was the active member of the pair.

Further evidence relating the P^{VV} allele directly to the promoter of *Ds* chromosome breakage was obtained from two other sources. Three near-homozygous lines, designated "4Co63" (P^{WW}), "22," and "23" (P^{WR}), were converted to their

P^{vv} counterparts by outcrossing to variegated pericarp and then backcrossing variegated pericarp segregates recurrently four times to the respective inbreds. On testing these two sets of approximately isogenic stocks for a promoter of *Ds* chromosome breakage, the standard inbred lines were found to be negative, whereas the corresponding variegateds were positive. Finally, the P^{vv} allele was introduced into a fourth inbred line known as "Hy." Pollen from seventeen plants following the second backcross mating ($P^{vv}\ P^{wR} \times P^{wR}$) was applied to appropriate *Ac*-testers. Fourteen of the plants proved to carry variegated pericarp. All of these gave a positive test for "activation" of *Ds*. The other three plants had colorless pericarp and were found to be negative in the same test.

Identification of the Endosperm Mottling with Ds Chromosome Breakage.—There are four lines of evidence supporting the conclusion that the endosperm mottling observed in testcrosses with variegated pericarp is due to *Ds* chromosome breakage: (1) linked marker genes in the short arm of chromosome 9 were lost concurrently; (2) mottled kernels of the predicted classes were formed with the frequencies expected if the breaks involved occurred between the waxy locus and the centromere, the region in which *Ds* is located; (3) the mottling pattern with reference to aleurone color differed characteristically between *I Ds/C ds/C ds* and *C Ds/c ds/c ds* endosperms; and (4) differences occurred in the mottling pattern between kernels, and between areas within kernels, related to the time and frequency of chromosome breaks. McClintock[3] has described these phenomena in detail as regular features of the *Ac-Ds* system. The data now to be presented demonstrate that like results are observed when Modulator, rather than Activator, is brought into association with *Ds*.

Pollen from an *Ac*-tester of the genotype *I Wx Ds/C Wx Ds*, no-*Ac*, *RR*, P^{ww} was placed on a *C Wx ds/c Wx ds*, *r r*, $P^{vv}\ P^{wR}$ plant. All the colored kernels in the resulting ear, designated "32-6-1," showed *R*-spotting. Some of them also appeared to display one or the other of the two types of mottling expected from *Ds* chromosome breakage. The colored areas in kernels presumed to be *I Ds/C ds/C ds* were uniformly pigmented except for the above mentioned *R*-spotting. The kernels tentatively classified as *C Ds/c ds/c ds*, on the other hand, had a different phenotype. The colored areas were not solidly pigmented but were "perforated" with colorless patches. This pattern is typical of *C* to *c* changes. Family 32-129 was grown from kernels selected as probably showing *I* to *C* mottling. Five plants, Nos. 1, 4, 6, 15, and 16, were selfed. All were homozygous nonwaxy and gave 13:3 ratios of colorless to colored kernels. Their genotype, therefore, was *I Wx Ds/C Wx ds*, *R r*. Two other kernels on ear 32-6-1 which appeared to be showing *C* to *c* mottling gave rise to plants Nos. 3 and 4 in family 32-148. These two individuals likewise were homozygous nonwaxy but gave 9:7 ratios of colored to colorless kernels. Their genotype, therefore, was *C Wx Ds/c Wx ds*, *R r*.

Pollen from the seven designated plants in families 32-129 and 32-148 was place on *c wx ds*, no-*Ac*, *R R* individuals. The data obtained from these latter matings are summarized in Table 2. It will be noted that two of the progenies entered in Table 2, 32-129-16 and 32-148-4, contained no mottled kernels. The ears borne by the two staminate parents likewise proved to have nonvariegated pericarp. The most likely explanation for these discrepant cases is that they represent misclassifications of the parent hybrid seeds as a result of the difficulty of dis-

tinguishing regularly on individual kernels mottling due to *Ds* chromosome breakage in the presence of certain types of *R*-spotting.

Approximately equal numbers of colored and colorless kernels resulted from the testcrosses recorded in Table 2. This proportion is a consequence of segregation

TABLE 2

DISTRIBUTION OF KERNELS IN TERMS OF *Ds* CHROMOSOME BREAKAGE EFFECTS FOLLOWING THE MATINGS (*a*) *c wx ds* ♀ × *I Wx Ds/C Wx ds* ♂ AND (*b*) *c wx ds* ♀ × *C Wx Ds/c Wx ds* ♂

	Testcross Progeny				
	No. Kernels				
	Colorless		Colored		
F_1 Staminate Parent	*Wx* to *wx*	*Wx*	*Wx*	*C Wx* to *c wx*	Per Cent Colored with *Ds* Breaks
(1)	(2)	(3)	(4)	(5)	(6)
			(*a*) Segregating *I* and *C*		
32-129-1	76	123	179	10	5.3
-4	51	109	160	19	10.6
-6	62	149	180	27	13.0
-15	49	98	132	22	14.3
-16	No *Ds* mottling				
			(*b*) Segregating *C* and *c*		
32-148-3	22	173	111	81	44.2
-4	No *Ds* mottling				

of *I* and *C* in group *a* and of *C* and *c* in group *b*. Within the colorless class in group *a* two phenotypes occur. One shows *Wx* to *wx* mottling (col. 2), and the other is uniformly *Wx* (col. 3). Assuming, for the sake of argument, that Modulator is the promoter present of the *Ds* activity resulting in chromosome breakage, the first phenotype would be expected to occur as the result of fertilization of a *c wx ds*, no-*Mp*/*c wx ds*, no-*Mp* fusion nucleus in the female gametophyte of the pistillate parent by an *I Wx Ds*, *Mp* sperm. Breaks at *Ds* during development of the resulting endosperm would give *Wx* to *wx* mottling. The other three kinds of sperm (*I Wx Ds*, no-*Mp; I Wx ds*, *Mp;* and *I Wx ds*, no-*Mp*), comprising one noncrossover and two crossover classes, would not yield mottled kernels because one or the other, or both, of the two components necessary for chromosome breakage, *Ds* and *Mp*, would not be present.

Similarly, within the colored class in group *a*, some of the kernels are *Wx* throughout (col. 4), and the others show *C Wx* to *c wx* mottling. The latter phenotype is of particular interest in that it involves the simultaneous loss of the *C* and *Wx* marker characters, an event which points clearly to chromosome breakage proximal to the *Wx* locus as the causal factor. The *C Wx Ds*, *Mp* sperm functioning in the production of kernels of this kind is expected as a product of crossing-over in the *I Wx Ds/C Wx ds* staminate parent.

The same four phenotypes discussed above appear in the single family in group *b*, but in a different ratio. The colorless kernels showing *Wx* to *wx* mottling, entered in column 2, are expected as products of fertilization by *c Wx Ds*, *Mp* sperm, a crossover class. The more numerous kernels exhibiting the simultaneous loss of *C* and *Wx* (col. 5) are in accord with expectation for one of the parental types of male gametes.

A more critical analysis of the above data is possible if the frequencies with which the four classes of kernels on the testcross ears are taken into account. The postulated factors determining these frequencies are (1) the linkage of *Ds*

with the *wx* and *c* loci and (2) the independent assortment of a promoter of *Ds*, assumed in these stocks to be Modulator, a regular component of the variegated pericarp allele. The question at issue is whether, in stocks hemizygous for *Mp* rather than *Ac*, the proportions of the four classes of kernels observed conform to expectation on the the hypothesis that chromosome breakage is occurring at *Ds* in its standard position between the *wx* locus and the centromere.

McClintock[3] has found that *Ds* in its standard position is located four crossover units proximal to waxy. *C* and *wx*, according to Hutchison,[5] show 20–25 per cent recombination. *C* is distal to *wx*. The amount of crossing-over between *C* and *Ds*, applying the method of maximum likelihood to the values in Table 2, was found to be 26.4 per cent. The observed distribution tested against this estimate gave a total chi square of 26.0 for 14 d.f. Since the 0.05 probability figure is 23.7, the observed figures are just significantly different from the estimated. The largest departure from expectation was shown by family 32-129-1.

The discrepancy could be the result of misclassification of a few kernels for mottling versus nonmottling. This is a possible explanation, particularly in view of the fact that some mottling was detected on the kernels in these families involving phenotypic changes from *C* to *c* and *Wx* to *wx* not related to *Ds* activity. The affected areas were small and few in number on any one kernel but appeared on many kernels. In general, they could be distinguished from *Ds* effects on the basis of size and pattern, but occasionally mistakes in classification may have been made. Similar events affecting the *wx* locus have been demonstrated by Mei Wang Fradkin in this laboratory to occur in other strains not carrying *Ds* and to be greatly increased when P^{vv} is present.

The deviation in the total population might be explained also if the frequency of crossing-over in any one plant lay outside the usual limits as a result of transposition of *Ds* from its standard position, for example, or for other reasons. In order to evaluate this possibility, the frequency of crossing-over between *Ds* and *C* was estimated by the maximum likelihood method for each family in Table 2 separately. The five estimates obtained were 13.4, 25.7, 31.8, 30.0, and 21.0 per cent, respectively. On testing the observed frequencies against these values, the total chi square for 10 d.f. was 15.11, and the differences thus were nonsignificant.

It may be concluded from the analysis, therefore, that the data are in satisfactory agreement with a hypothesis that the observed endosperm mottling resulted from breaks in the region of the standard position of *Ds* in chromosome 9. Furthermore, the evidence bears out the postulate that one independently inherited promoter of *Ds* events was present in hemizygous condition. The fact that families 32-129-16 and 32-148-4, whose respective staminate parents did not carry variegated pericarp, failed to show *Ds* mottling, whereas the other five families, from variegated pericarp plants, gave positive results, points to the conclusion that this promoter of *Ds* corresponds to the P^{vv} allele.

Further data supporting these conclusions were obtained from 79 additional testcross ears of similar breeding to those in Table 2, except that the F_1 heterozygotes were *wx Ds/Wx ds*, so that the waxy character could not be employed as a marker. The P^{vv} alleles represented were derived from five geographical sources. A summary of the results follows.

None of the seeds showed *Ds* mottling on 33 of the 79 ears. These cases of misclassification of the mother-kernels represent either overlaps in the *R*-spotted and *Ds*-mottled phenotypes or chance breakage events, which are not rare in the endosperm. Thirty-six of the remaining families showed *C* to *c* mottling, and ten gave *I* to *C* mottling.

The amount of crossing-over between the *Ds* and *C* loci was estimated by the maximum likelihood method from the remaining 46 families, plus those in Table 2, to be 28.5 per cent. Accepting this figure and a probability of 0.5 that each seed would receive Modulator, as a promoter of *Ds* chromosome breakage, the expected proportion of *C*/*c* kernels showing mottling is about 36 per cent and of *I*/*C* kernels approximately 14 per cent. The observed average percentages for the two groups were 33 and 10, respectively. These results, therefore, are in substantial agreement with the data in Table 2 in showing that mottled kernels occur with the frequencies to be expected from the action of a *Ds* unit located in its standard position. It was observed also, in the majority of cases in which the breakage event occurred early enough in endosperm development to permit scoring for shrunken, that *Sh* and *C* were lost together.

Seven of the 46 segregating testcross families contained *Ds*-mottled kernels even though the respective staminate parents had nonvariegated pericarp. Occasional progenies of this kind are to be expected as a result of transposition of Modulator from the *P* locus to another site (Brink and Nilan[2]).

Irregularities in distribution of phenotypes, not calling for qualification of the main conclusion, were observed in some families. Most of these deviations probably resulted from early breaks at *Ds*, leading to inviability of the pollen carrying the deficient chromosomes, a phenomenon which McClintock[6] also has observed.

Modulator as the Component of the P^{VV} Allele Promoting Ds Breaks.—The evidence presented in the two preceding sections shows that variegated pericarp acts as a promoter of *Ds* chromosome breakage. Assuming the validity of the hypothesis of Brink and Nilan[2] that the P^{VV} allele is a compound structure, comprising the P^{RR} gene for red pericarp and cob conjoined with Modulator, the question may now be asked: Which of the two components is the "activator" of *Ds*, or are both involved? The P^{RR} gene can be excluded on two grounds. As the data in Table 1 show, all the P^{RR} stocks tested for a promoter of *Ds* chromosome breakage gave negative results. Second, plants not carrying P^{RR}, either as such or as a component of the variegated allele, nevertheless may promote *Ds* chromosome breakage. Seven such individuals were found among the 46 testcrossed heterozygous waxy plants in the families discussed earlier which were segregating P^{VV} and either P^{WR} or P^{WW}. These plants are presumed to have carried transposed Modulator, that is to say, a Modulator unit which has shifted position from the *P* locus to another site in the genome. Independent evidence for the presence of transposed Modulator in these particular individuals is not available, but the frequency observed is of the same order as transposition in stocks which have been tested directly (cf. Brink and Nilan[2]). These considerations point to Modulator as the promoter of *Ds* chromosome breakage in the present strains.

Activator and Modulator show striking dosage actions. These have been studied independently by McClintock[3] for *Ac* in terms of the timing of *Ds* events and by Brink[4] for *Mp*, using the variegated pericarp phenotype as the criterion.

An effort was made in the present study to relate the endosperm mottling resulting from *Ds* chromosome breakage to Modulator dosage. It was found that most of the results could be accounted for on the assumption that a single *Mp* unit was present. The occurrence on some mottled kernels of adjacent areas showing sharp differences in timing of the loss of marker characters suggested that *Mp* has dosage effects on *Ds* similar to those of *Ac*. Attempts to correlate grade of variegated pericarp with endosperm mottling pattern, however, were generally unsatisfactory, and the results of this part of the study are not reported. The difficulties probably arose from two main sources. In the first place, the P^{vv} alleles used varied widely in origin and differed inherently in phenotypic expression. Some gave relatively coarse variegation, others a finer pattern, on similar genetic backgrounds. This fact suggests that the Modulator unit itself varies, as the "state" of *Ac* has been observed to vary. Second, variegated pericarp fluctuates greatly, depending on the residual inheritance. The procedures followed did not provide for control of this variation. A separate study is in progress in which the effect of these extraneous factors should be reduced to a level at which the dosage effects of Modulator on *Ds* chromosome breakage can be determined.

Discussion.—The results of this investigation show that a promoter of *Ds* chromosome breakage is present in maize plants possessing the variegated pericarp allele, P^{vv}, and in some nonvariegated segregates from heterozygous variegated parents. The evidence strongly suggests that the promoter is Modulator, an element which Brink and Nilan[2] have postulated as accessory to the P^{RR} gene, conditioning red pericarp and cob, in forming the unstable P^{vv} allele and which, from time to time, undergoes transposition to other locations in the genome. It is found that, with reference to *Ds* chromosome breakage, Modulator is closely parallel to Activator, which McClintock[3] discovered in an unrelated stock which had recently undergone various changes in chromosome structure as a result of the operation of the chromosome type of breakage-fusion-bridge cycle.

Brink and Nilan[2] concluded from their study of the variegated pericarp allele that Modulator is a "mutating agent" at the *P* locus, similar to *Ds* at or near other loci. Brink[4] found that transposed Modulator has dosage effects on variegated pericarp like those of *Ac* on *Ds* events. Thus *Mp* combines some, at least, of the properties of both *Ac* and *Ds*. Modulator may be a unitary element possessing attributes of both *Ac* and *Ds*, or it may be bipartite. The properties of all three elements must be more fully determined before definitive conclusions can be drawn concerning the homologies between Activator, Dissociation, and Modulator. McClintock[3] has established the role of *Ac* as the co-agent of *Ds* in conditioning instability and chromosome breakage at the locus at which *Ds* is inserted. Unpublished data, more recently obtained by McClintock[6] and mentioned here with her permission, show that *Ac*, in the absence of *Ds*, renders the bronze locus unstable when located adjacent to it. Thus *Ac*, as an autonomous unit, appears to have effects on stability at the bronze locus corresponding to those of *Mp* at the *P* locus.

Summary.—It is shown that Modulator (*Mp*), postulated by Brink and Nilan as a component of the long-known variegated pericarp allele (P^{vv}) in maize, corresponds to Activator (*Ac*) a unit which McClintock identified as the co-agent

of Dissociation (*Ds*) in causing instability and chromosome breakage at the *Ds* locus. The relationship was established by showing that Modulator, like Activator, promotes *Ds* chromosome breakage at the standard position of Dissociation in the short arm of chromosome 9. Four classes of stable *P* alleles tested and another highly unstable allele, mosaic pericarp (P^{MM}), did not promote *Ds* chromosome breakage. The evidence shows that Modulator has certain properties in common with both Activator and Dissociation.

The writers desire to express their appreciation to Dr. Barbara McClintock for generous assistance in this study. She supplied the *Ac*-tester strains used, aided in planning certain experiments, verified classification of part of the material, and reviewed the present manuscript. Newton Morton assisted in determination of the crossing-over frequencies by the maximum likelihood method. We are indebted to Dr. P. C. Mangelsdorf for seed of various Latin-American strains of maize from which certain *P* alleles were extracted. Professor R. I. Brawn, of Macdonald College, Quebec, kindly supplied the original Q36 stock.

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AN ALLELE OF THE S(s) BLOOD GROUP GENES

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In 1953 Wiener, Unger, and Gordon[1] found a new blood group antibody, anti-U, in the serum of an American Negress. Wiener, Unger, and Cohen[2] reported that anti-U agglutinated the red cells of all 1,100 Caucasians tested but that it failed to agglutinate the red cells of 12 out of 989 Negroids. These investigators tested the blood of relatives of four of the twelve negative persons and concluded: "Presumably, the factor is inherited by a pair of allelic genes *U* and *u*, where gene *U* determines the presence of the factor and gene *u* its absence." It was considered most probable that *U* was dominant to *u*, in which case the gene frequencies in New York Negroes were *U* 89 per cent and *u* 11 per cent, and the genotype frequencies were *UU* 79.2 per cent, *Uu* 19.6 per cent, and *uu* 1.2 per cent. The authors suspected an association with the MNSs system when they noted: "The

peculiar racial distribution suggested the possibility that the factor *U* might be an alternate of the Hunter factor or Henshaw factor"; this, however, they were able to exclude. (The Hunter and Henshaw antigens[3, 4] are associated with the MNSs system in a way not yet precisely understood.)

We have been able to show that there is indeed an association between U and the MNSs system: *uu* homozygotes lack both S *and* s antigens. This observation followed the identification of a second example of anti-U.

A Second Example of Anti-U.—In the serum of an American Negress, Mrs. Q., we found, besides anti-N and anti-D, a third antibody which sensitized the red cells of all M, dd Caucasians tested. It was suspected that this third antibody was anti-U when Mrs. Q.'s red cells were not agglutinated by the original anti-U serum; it was established when a sample of *uu* blood (J. W.; sent from New York) failed to absorb the antibody, which was readily removed by five Caucasian U samples.

uu and the MNSs System.—The two *uu* samples, from Mrs. Q. and J. W., were tested with all the blood group antibodies available to us, but their reactions with those of the MNSs system alone were noteworthy; they are summarized in Table 1. These most unexpected negative results with both anti-S and anti-s were given by all four anti-S and both of two anti-s sera available; they were confirmed by absorption tests as shown in Table 2.

TABLE 1

THE REACTIONS OF TWO *uu* BLOOD SAMPLES WITH THE ANTIBODIES OF THE MNSs SYSTEM*

	Anti-M	Anti-N	Anti-S	Anti-s	Anti-U
Mrs. Q.	+	−	−	−	−
J. W.	+	+	−	−	−

* Neither sample was agglutinated by anti-Hu, anti-He, or anti-Vw, three further antibodies associated with the MNSs system (see Landsteiner, Strutton, and Chase, *op. cit.*; Chalmers, Ikin, and Mourant, *op. cit.*; and Mia Van der Hart, Hélène Bosman, and J. J. van Loghem, *Vox sanguinis*, **4**, 108–116, 1954).

TABLE 2

CONFIRMATION THAT *uu* BLOOD (J. W.) LACKS THE ANTIGENS S AND s*

	Dilutions of Serum					
	1/1	1/2	1/4	1/8	1/16	1/32
Against SS red cells:						
Anti-S unabsorbed	+++	+++	++	w	−	−
Anti-S absorbed × *uu*	+++	++	++	(+)	−	−
Anti-S absorbed × *ss*	+++	+++	++	(+)	−	−
Anti-S absorbed × *Ss*	−	−	−	...	...	...
Against ss red cells:						
Anti-s unabsorbed	+++	+	+	(+)	(+)	(+)
Anti-s absorbed × *uu*	+++	++	++	w	w	w
Anti-s absorbed × *ss*	−	−	−	−	−	−

* In each absorption an equal volume of serum and packed red cells was used.

The most obvious interpretation is that *u* is an allele of *S* and *s*, in which case S^u is a more appropriate symbol. The possibility that MS^u and NS^u are chromosomes analogous to D— in the Rh system[5] is perhaps worth considering, though this possibility will be ruled out when and if anti-S^u is discovered. In any case the specificity of anti-U can be expressed as "anti-Ss," implying that the antibody reacts both with the antigen S and with the antigen s.

No example of blood negative with both anti-S and anti-s has been found in testing about 1,000 white people,[6] and this, together with the 1,100 tests of Wiener, Unger, and Cohen,[2] using anti-U alone, shows that the gene S^u must be lacking, or very rare, in Caucasians.

Future progress in the study of the allele S^u will surely depend on the testing of American Negro families. So far, only 145 Negro bloods have been tested with both anti-S and anti-s (Levine, Kuhmichel, Wigod, and Koch[7]); none was negative with both, but, since we now know that about 20 per cent must be heterozygous SS^u or sS^u, they will have been misclassified as SS or ss. It is possible that titrations of anti-S or anti-s might help to distinguish these hidden heterozygotes from the true homozygotes.

Some Observations concerning Anti-U.—Absorption with various Caucasian bloods showed that Mrs. Q.'s serum contained anti-B, anti-N, and anti-D, as well as anti-U. Happily, the *uu* red cells of J. W. contained the antigens B, N, and D and were successfully used to remove all but anti-U from the serum of Mrs. Q.

We have said that anti-U can be described as anti-Ss, and this is certainly the easiest way to think of it. However, preliminary titration, absorption, and elution tests using SS and ss red cells have failed to split Mrs. Q.'s anti-U into anti-s and anti-S. This hardly surprised us, remembering the difficulty of demonstrating the presence of anti-c and anti-C when they occurred in one serum.[5] We suppose that Mrs. Q.'s anti-U is immune—it is certainly an incomplete antibody; we guess it has been stimulated by fetal s antigen.

The Family of Mrs. Q.—Samples of blood were obtained from the husband of Mrs. Q. and from five of their children. The reactions with antibodies of the MNSs system were as shown in the accompanying tabulation. At the time of

	Anti-M	Anti-N	Anti-S	Anti-U
Husband	−	+	−	+
First child	+	+	−	+
Second child	+	+	−	+
Third child	+	+	−	+
Fourth child	+	+	−	+
Fifth child	+	+	..	..

testing these members of the family, the relation of U to S and s had not been realized, and none of them were tested with anti-s. We hope to be able to retest these children with sufficient controls to show whether or not sS^u gives a single-dose reaction with anti-s. Mr. Q., like Mrs. Q., was negative with anti-Hu and anti-He.

The family, with the exception of the fifth child, was tested for all the other eight established blood group systems and for a number of "private" antigens. Mrs. Q., who has never been transfused, is *cde/cde* and has anti-D in her serum as well as anti-U (and anti-N). Her fifth child died of hemolytic disease 8 hours after birth. This child was *cDe/cde*, like the first four children, and presumably sS^u, so it is not possible to say whether the anti-U contributed to the disease.

Summary.—A blood group system defined by anti-U has recently been described by Wiener, Unger, Gordon, and Cohen. Anti-U reacted with the red cells of all Caucasians tested and with the red cells of about 99 per cent of American Negroes—the remaining 1 per cent were assumed to be of the genotype *uu*.

During the investigation of a second example of anti-U, we have found that the red cells of *uu* persons do not react with either anti-S or anti-s. The most obvious

explanation is that *u* is an allele at the *Ss* locus of the MNSs system, and we suggest that S^u is an appropriate symbol for this allele. Anti-U can most easily be thought of as anti-Ss.

We are grateful to many people for the antisera used in this investigation but would particularly like to thank Dr. A. S. Wiener for a sample of the original anti-U. We are very much indebted to Dr. Lester J. Unger for samples of blood from the *uu* donor, J. W.

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*CHROMOSOME SHATTERING BY ULTRAVIOLET RADIATION (2650 A)**

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Introduction.—The mutagenic action of ultraviolet radiation in the region near wave-length 2650 A and the strong absorption of this region by constituents of the chromosomes make morphological changes in the chromosomes following exposure to this wave-length of special interest.

In a study of the modifying effects of pretreatment with ultraviolet radiation on gamma-ray–induced aberrations in *Tradescantia* pollen tube chromosomes,[1] shattering of portions of chromosomes was observed in some of the ultraviolet control tubes grown from pollen which had received very high doses of 2650 A radiation. Portions of one or more of the chromosomes were broken into a number of small fragments, whereas the rest of the chromosome set contained few or no breaks.[2] "Shattering," as used in this paper, refers to this localized multiple fragmentation of chromosome parts.

Direct and indirect studies of ultraviolet radiation effects on the morphology of chromosomes have been made. Ultraviolet-induced gross morphological changes have been described, but no shattering of chromosomes by ultraviolet has been previously reported. Since shattering was so striking in the affected chromosomes, a further study of this phenomenon was undertaken. The experiments reported in this paper are the result of this study and of considerations concerning the relation between incident dose of 2650 A radiation and chromosomal effects.

Materials and Methods.—Pollen from Sax's clone 5 of *Tradescantia paludosa* was desiccated over anhydrous $CaSO_4$ for about 1 hour, then spread in monolayers on glass plates, and sealed in dry Lucite boxes fitted with optical-grade fused

quartz windows. Two samples were inclosed in each box. One of these was exposed to ultraviolet radiation through the quartz window; the other was covered by black paper and grown as a control. Pollen for some of the experiments was collected from greenhouse plants; for other experiments it came from field-grown plants which had been transferred to nutrient solutions and brought to flowering under controlled conditions. Only plants derived from cuttings of a single plant of clone 5 were used, eliminating genetic variability.

Irradiations were carried out at room temperatures (25–30° C.). Immediately after irradiation the treated and control pollens were planted on Bishop's colchicinized agar-lactose medium, according to Conger's[3] technique. After 20 hours at 22° C. the tubes were fixed in acetic acid–alcohol and stained by the Feulgen method, and metaphase nuclei were scored for aberrations.

The source of energy for irradiation was a water-cooled capillary mercury arc of the Daniels-Heidt type, operating at 3 amperes and 100 volts. Monochromatic radiation was isolated from the spectrum of the arc by means of a large crystal quartz monochromator. The intensity of the energy incident on the plane of the pollen was measured by means of a thermopile and a sensitive galvanometer calibrated against a standard lamp from the United States Bureau of Standards. Energy readings were taken at the beginning and at the end of each exposure and the incident energy computed from the average. No spectroscopic analysis of the purity of the radiation was made. However, other investigators using this same equipment under the same conditions find contamination by other wave-lengths to be less than 2 per cent of the total energy in the emergent beam.

Incident doses of from 10^4 to 4.5×10^6 ergs/cm^2 of 2650 A radiation were delivered to monolayer samples of dry pollen, and four different doses were used in each experiment. This dosage range begins somewhat below the 90 per cent lethal dose for most microörganisms, includes the optimum dose (1.6×10^6 ergs/cm^2) for maximum production of mutations in maize pollen,[4] and exceeds the maximum dose tolerated by maize pollen. Doses of 10^6 ergs/cm^2 of 2804, 3130, and 3650 A radiation were also given to samples of pollen, and higher doses of 3650 A wave-length were also used. Results with the longer wave-lengths will be published elsewhere, but no shattering occurred following the doses used.

Precautions against Ozone Contamination.—Wangermann and Lacy[5] warn against confusing the effects of the ozone produced by short-wave-length ultraviolet with the effects of the radiation itself. Conger[6] has found that ozone breaks pollen chromosomes. In these experiments the radiation chamber was sealed to prevent contamination by stray ozone in the radiation room. The quartz window was exposed to the monochromatic beam only, and 98 per cent of the energy in the beam was 2650 A radiation, which is not absorbed by oxygen and does not produce ozone at atmospheric pressures. The control pollen which was inclosed in each chamber served as a check against effects which might be due to changes in the atmosphere of the chamber. Kirby-Smith and Craig[7] have shown that breakage of pollen chromosomes by 2650 A radiation occurs in the absence of oxygen.

Precautions against Photorecovery.—The pollen was covered during the interval between treatment and planting. It was dusted on the surface of the medium in a dimly lit constant-temperature room, and the tubes were grown in darkness, so that a minimum of light in the photorecovery range reached the cells after treatment. Since all plantings were carried out under approximately the same

conditions of light, the results are probably not affected by photorecovery phenomena. Kirby-Smith and Craig[7] have found no photorecovery of irradiated pollen.

Intensity Changes.—Within a given experiment the dose was varied by varying the time of exposure; the only intensity changes were those due to fluctuations in the arc itself. However, from one experiment to another, the intensity varied by as much as a factor of three. Swanson[8] and Kirby-Smith and Craig[7] found no intensity effect in their studies of the effects of ultraviolet on pollen tube chromosomes.

Observations and Results.—Abnormalities observed in the metaphase chromosomes following 2650 A radiation include abnormal coiling, seen in every chromosome in some cells and in none of the chromosomes in other cells; stickiness of chromosomes or of the chromatids; lesions (i.e., unstained portions) of chromosomes or of chromatids; and constrictions (or thin places) in the chromatids.

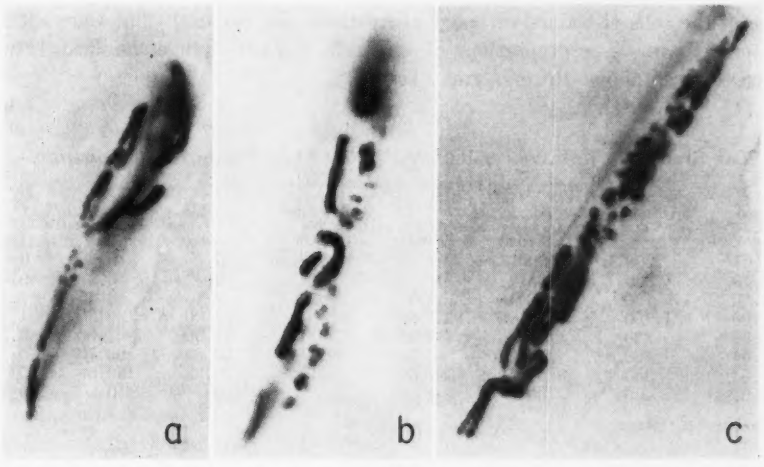

FIG. 1.—Photomicrographs of *Tradescantia* pollen tube chromosomes after exposing pollen to 2650 A radiation. × 4,200. Dose (× 10^6 ergs/cm^2): *a*, 1.76; *b* and *c*, 4.55. *a:* Shows a chromatid deletion in one arm and shattering in the other arm of one chromosome, also shattering in one arm of another chromosome. *b:* The proximal portion of one arm of one chromosome is shattered, one arm of another is shattered into oriented fragments, and one chromosome is normal. *c:* Two chromosomes are normal, and portions of the others are broken into ragged unoriented fragments.

Constrictions usually involved both chromatids and gave the chromosome a lumpy or "beaded" appearance. They were more frequently seen at the ends of chromosomes and usually involved only a portion of a chromosome. Similar abnormalities are found in pollen tube chromosomes following a variety of treatments, but they follow no consistent pattern and are frequently seen in untreated tubes. In these experiments no attempt was made to score them. Metaphase chromosomes were not shortened by the treatment.

The most striking effect of high doses (10^6 ergs/cm^2 and above) was a shattering of chromosomes. Only a portion of one chromosome was affected in some cells and all of the chromosomes in a few cells. Both chromatids were nearly always involved in an affected area (Fig. 1, *a*, *b*). Typically, these "shatters" remained in line (Fig. 1, *b*), but sometimes they were not oriented with reference to one

another (Fig. 1, *c*). The extent of shattering in an affected chromosome varied from a few fragments to a cloudlike mass of minute irregular particles which were sometimes mixed with larger fragments. Some of the fragments had rounded ends, but more frequently they were somewhat ragged. Extensive shattering of affected areas was observed more frequently following the higher doses. Shattered portions were sometimes found in combination with the more general abnormalities described and sometimes in combination with the ordinary aberrations of pollen tube chromosomes.

Fragments were scored as chromatid deletions (cd) if they were unpaired or of unequal length and the ends rounded. They were scored as isochromatid deletions (iso) if they were paired or of equal length and the ends rounded. Fragments with ragged edges were simply scored as shatters. Results of the scoring are shown in Tables 1 and 2 and in Figure 2. Chromatid and isochromatid deletions occurred with significant frequencies, and a few exchanges (exch) were observed. Some of the cells contained ordinary aberrations (cd, iso, exch) but were without shattered parts. The proportions of such cells did not change significantly with increased dose above 10^6 ergs/cm^2 (Table 1).

TABLE 1

THE EFFECTS OF ULTRAVIOLET RADIATION (2560 A) ON *Tradescantia* CHROMOSOMES

Dose ($\times 10^6$ Ergs/Cm^2)	Number of Cells Scored	Percentage of Cells: Normal	With Shatters	With Aberrations but No Shatters	No. of Aberrations per 100 Cells*: cd	iso	exch
0.01	200	100	0	0	0	0	0
0.10	300	95	0	5	2.3	3.3	0
0.50	750	89.18	2.51	8.31	8.16	4.45	1.13
1.00	1,150	62.00	23.60	14.40	23.00	29.00	1.14
1.75	200	55.00	38.00	7.00	15.00	67.00	0.50
2.64	300	36.00	57.30	6.70	20.30	96.70	0
4.55	350	34.69	56.66	8.65	26.33	139.69	0
Control	400	100	0	0	0	0	0

* Exclusive of shatters.

TABLE 2

RELATION BETWEEN DOSE AND EXTENT OF CHROMOSOMAL DAMAGE BY 2650 A RADIATION

Dose ($\times 10^6$ Ergs/Cm^2)	Number of Cells Scored	Percentage of Cells: Normal	Containing Less than 10 Fragments*	Containing More than 10 Fragments	Percentage of Chromosomes with Breaks
1.00	700	64.5	21.7	13.8	15
1.76	200	55.0	11.5	33.5	22
2.64	300	36.0	16.1	47.9	27
4.55	350	34.7	15.0	50.3	29

* Fragments include shatters and other aberrations.

Discussion.—The oriented fragments observed in these experiments suggest a possible relation between shattering and the ultraviolet-induced morphological changes in grasshopper chromosomes described by Carlson and Hollaender.[9, 10, 11] They found that living chromosomes in prophase stages become thinner immediately after exposure to low doses of 2537 A radiation and are finally replaced by fine granules as they undergo temporary reversion to earlier stages.[9] Following high doses of 2537 or 2650 A, prophase chromosomes change "from threads of uniform diameter to a moniliform shape" during reversion; and prometaphase, metaphase,

and anaphase chromosomes "appear blurred and indistinct."[10] High doses of 2537 A radiation cause "breakdown of the prometaphase and metaphase chromosomes to form strings of coarse granules."[11] No detached fragments were observed, but the cells were followed for only one division. The "moniliform" threads and "strings of coarse granules" could represent potential shattering.

Swanson[8, 12] found that ultraviolet radiation produced chromatid deletions in *Tradescantia* pollen tube chromosomes when the growing tubes were irradiated. In his experiments no other types of aberrations were produced; metaphase chromosomes were decidedly shortened by 2537 A radiation but not by a mixture of 2967 and 3022 A, and the dose-break relation was linear. Results of the experiments reported in this paper include the production of both chromatid and isochromatid deletions and a few exchanges, localized shattering of chromosome parts after high doses of 2650 A, metaphase chromosomes which are not shorter than control metaphases, and a dose-break relation which does not appear to be linear. The two sets of experiments differ in the stages irradiated, the wavelength used, and the dose range covered. These differences in experimental conditions may account for the differences in results.

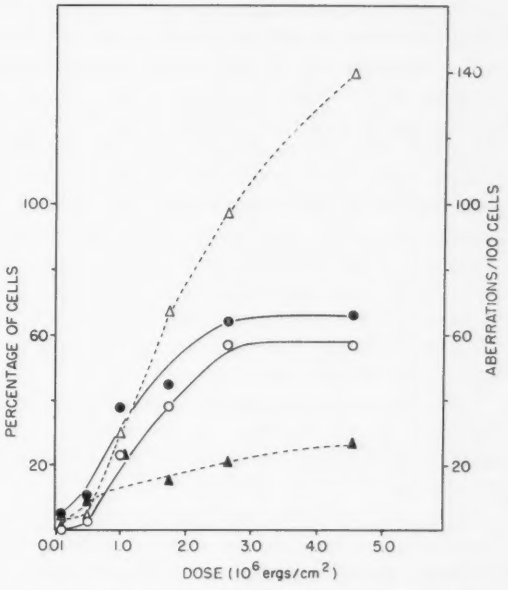

FIG. 2.—The effects of different doses of 2650 A radiation on *Tradescantia* pollen tube chromosomes. *Solid lines* represent percentages; *broken lines*, aberrations per 100 cells. *Open triangle* = iso deletions; *solid triangle* = cd deletions; *solid circle* = cells with breaks; *open circle* = cells with shatters.

The shattering effect following doses of 10^6 ergs/cm^2 and more makes scoring of chromatid and isochromatid deletions difficult; below this dose the relation to dose appears to be nonlinear (Fig. 2). Both the fraction of cells containing breaks and the proportion of cells containing shattered chromosomes level off with increased dose. The percentage of cells containing more than ten fragments also appears to reach a saturation point, as does the proportion of affected chromosomes (Table 2). The curve for isochromatid deletions rises rather sharply with increased dose. However, under the system of classification used in scoring, many of the shattered pieces could be scored as isochromatids at higher doses (shattering nearly always involved both chromatids), and this curve cannot be interpreted as significant. Evaluation of these data for dosage effects must take into consideration the fact that only those cells which reached metaphase were

scored. There may have been a differential dosage effect on germination or on division, though there was no evidence of this.

Quantitative interpretations of the data in terms of the actual energy reaching the nuclei are not possible. Nothing definite is known of the transmission of ultraviolet radiation by the parts of the dry pollen grain of *Tradescantia*. By use of the transmission data of Uber[13] for the wall of corn pollen and of Caspersson[14] for the cytoplasm of *Tradescantia* pollen mother-cells, if similar transmission for the wall and cytoplasm of dry grains is assumed, it is possible to estimate the amount of energy reaching a particular depth in the pollen grain. However, this would not give a true estimate of the energy reaching the chromosomes. The generative nucleus, i.e., the nucleus whose chromosomes are later scored, is elongate and curves around the spherical tube nucleus in the shape of a crescent. Different portions of it are at different depths in the ellipsoidal pollen grain, so that the actual energy reaching a given portion of this nucleus depends on its orientation with reference to the incident beam of energy. In a layer of randomly oriented pollen, different nuclei and different portions of the same nucleus receive different amounts of energy, the actual dosage depending on absorption by overlying materials. Portions of nuclei covered only by the wall of the grain and a thin layer of cytoplasm receive larger doses than portions also covered by the tube nucleus (highly absorptive) or by other portions of the generative nucleus or by both. Thus differential dosage within a nucleus could cause the localized shattering effect and the different degrees of shattering in an affected area. Chromosomes, or parts of chromosomes, which receive large amounts of energy could be severely affected, whereas those which receive little or no energy (because of absorption in the pathway) could be little affected.

These considerations lead to the suggestion that the shattering effect of the ultraviolet is a direct one on some constituent of the chromosomes at the time of irradiation and that only those parts which receive sufficient amounts of radiation are affected.

At the time of irradiation the chromosomes of the generative nucleus are greatly elongated, Feulgen-positive double threads within the slender crescent-shaped nucleus. Both the protein and the nucleic acid portions of chromosomes absorb 2650 A radiation.[15] Proteins undergo denaturation when irradiated with 2537 A,[16] and nucleic acids undergo depolymerization.[17] These in vitro studies of the action of ultraviolet radiation on constituents of the chromosomes are highly suggestive, but their relations to phenomena occurring in living cells under ultraviolet radiation need further study.

The results of 2650 A radiation on *Tradescantia* pollen chromosomes can be interpreted in terms of random orientation of the pollen grains at the time of irradiation and differential absorption by overlying materials, if direct action of the ultraviolet on chromosomal materials is assumed.

Summary.—Dry pollen of *Tradescantia paludosa* can be used for studying the effects of ultraviolet radiation on chromosomes. When irradiated with energy of 2650 A wave-length, chromatid and isochromatid deletions and some exchanges are produced. Shattering of chromosomes into many small fragments occurs after high doses of 2650 A radiation.

Drs. J. S. Kirby-Smith and A. D. Conger, of the Oak Ridge National Laboratory, and Dr. M. R. Zelle, of Cornell University, are largely responsible for the dosage determinations. Gratitude is hereby expressed to them and to Dr. C. P. Swanson of the Johns Hopkins University for invaluable suggestions, and to Miss Helen Johnston, who also helped with the experiments.

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THE NATURE OF THE PIGMENT INDUCED BY CHROMOGENIC INDUCTORS OF SERRATIA MARCESCENS

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In a previous report[1] it was demonstrated that some of the color variants of *Serratia marcescens* release chromogenic inductors with which other variants grown in the same medium can react to produce pigment. The induced pigment from these interactions varies from pink to a deep red resembling the color of the normal HY strain. Wrede[2, 3] isolated and analyzed the red pigment of *Serratia*, prodigiosin, which is structurally related to the porphyrins. The present study was undertaken to determine whether the pigment induced by the interaction of variants is the same as the pigment of the normal strain.

Pigment was isolated from the normal HY strain; from the variants W_1 (white), P_2 (pink), P_{18} (pink) and O_3 (orange); and from the interaction of P_2 with W_1 and P_{18} with O_3. Both P_2 and W_1 behave as inductors as well as reactors in relation to each other, and each of them forms red pigment when they are grown together. Variant O_3 reacts to the P_{18} inductor and produces red pigment, whereas the color

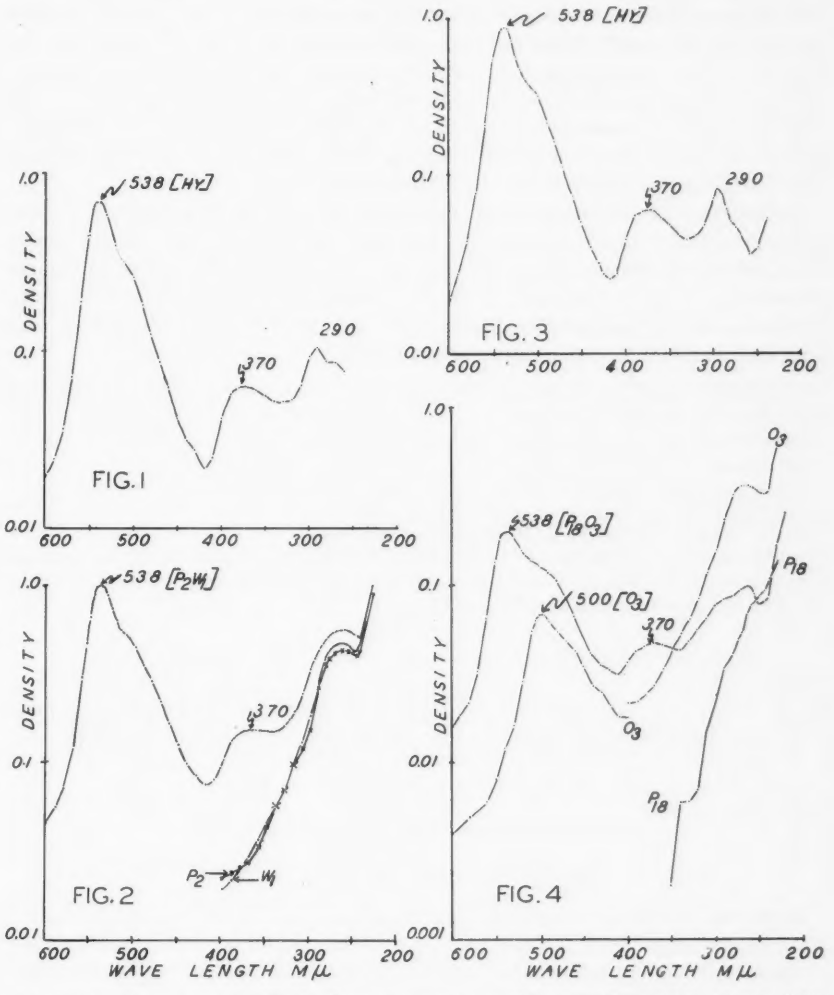
of P_{18} is unaltered by the presence of the O_3 variant. Cultures were grown on peptone-glycerol agar medium, as described previously, for three days at 25° C. Three Petri dishes were used for each extraction, and all cultures within each experimental group were grown and extracted simultaneously. The induced pigment was obtained by spreading a mixture of an inductor and a reactor variant on the same Petri dish.

The method of extraction was adapted from that described by Wrede.[2] The organisms removed from the surface of the Petri dishes were lysed in 20 ml. of 10 per cent KOH (treated for 20 minutes) and precipitated with 30 ml. of ethyl alcohol. The mixture was extracted twice with petroleum ether, 20 ml. each time. The ether extract was washed several times with ethyl alcohol and H_2O, filtered, and evaporated at room temperature. The residue in the evaporation dish was rinsed twice with distilled H_2O and eluted with 2 ml. of acidulated alcohol (95 ml. EtOH and 5 ml. 1 *N* HCl). The extracts were sealed in ampules until needed, at which time they were diluted with acidulated alcohol. Absorption curves were made with a Beckman spectrophotometer from 220 mμ to 600 mμ.

Experiment 1 included the normal HY strain, pure P_2, pure W_1, and the mixture of P_2 and W_1 grown together. The absorption curve for the extract obtained from the normal strain is given in Figure 1. There is a sharp peak at 538 mμ, with a slight shoulder at 500–515 mμ, and there are two peaks in the ultraviolet, one broad peak with its maximum at 370 mμ and a sharper peak at 290 mμ. The same peaks are displayed by the extract of the normal strain isolated in experiment 2 (Fig. 3). Neither of the extracts from the pure variants, P_2 and W_1, has an absorption peak at 538 mμ, but the induced pigment obtained from the mixed culture P_2W_1 has a peak at 538 mμ, with a slight plateau between 500 and 515 mμ similar to the absorption curve of the HY strain in the visible (Fig. 2). The P_2W_1 curve has a peak at 370 mμ resembling the curves of the normal HY, while the pure mutants lack this character. The curve of the normal strain has a third peak at 290 mμ, but this peak is absent in the P_2W_1 curve. Rather, this region of the P_2W_1 curve is similar to both of the pure mutants P_2 and W_1, which show broad peaks with maxima at 256–258 mμ. This region of the curve may represent the presence of the P_2 or W_1 substances covering the absorption which would be analogous to the 290 mμ peak of the HY strain.

Figure 4 presents the absorption curves for the pigment extracts obtained from O_3, P_{18}, and a combined culture of O_3 and P_{18} (experiment 2). The O_3 curve has a peak at 500 mμ and a broad peak in the ultraviolet with a maximum at 269 mμ. There are no specific peaks in the curve of the P_{18} extract either in the visible or in the ultraviolet. The pigment induced by combined growth of P_{18} and O_3 has a peak at 538 mμ and one at 370 mμ, in these respects resembling the curves of the normal pigment, while differing markedly from the curves of the extracts of the pure variants. The third maximum in this curve is found at 265 mμ. The broadening of the hump found in the $P_{18}O_3$ curve relative to the shoulder found in the curve of the HY strain at 500–515 mμ may be due to the presence of the pigment of the pure O_3 strain. No interference is expected from excess pure P_{18}, which has no absorption in the visible. Variant P_{18} does not form red pigment in the presence of O_3; thus the red pigment isolated from the mixed culture $P_{18}O_3$ is the change in O_3 in response to P_{18}.

The absence of pigment in the extracts of the pink cultures, P_2 and P_{18}, may be due either to low concentration or to insolubility in petroleum ether. The pink pigment of the P_{18} variant remains associated with the cellular debris and is not



FIGS. 1–4.—Experiment 1: Fig. 1, absorption spectrum of the petroleum ether extract of the HY strain of *Serratia marcescens;* Fig. 2, spectra of the extracts of the pure cultures of P_2 and W_1 and the combination P_2W_1. Experiment 2: Fig. 3, spectrum of the HY strain; Fig. 4, spectra of P_{18}, O_3, and $P_{18}O_3$.

obtainable in the petroleum ether fraction like the normal red pigment. Perhaps the pink pigment of *Serratia* variants is a different type of pigment from prodigiosin, or it may be bound to the structural elements of the cells so that it is not extractable by the method employed.

Prodigiosin is the only pigment of *Serratia* that has been subjected to chemical analysis.[2, 3] The absorption spectrum of prodigiosin as isolated by Wrede has been presented by Ehrismann and Noethling.[4] It has been claimed[5] that the chemically prepared pigment from the ether extract as obtained by Wrede can be further resolved into two components by column chromatography. The absorption spectrum of the O_3 extract in the visible is similar to the spectrum of one of the components of the normal pigment separated by chromatography and has a peak at 500 mμ.[6] If such a component is separable from an induced pigment such as that obtained from the combination of P_2 and W_1, neither of which has an absorption peak in the visible range, then the synthesis of the pigment of O_3 and the red pigment prodigiosin may be related by a common step.

Summary—The petroleum ether extracts of the normal HY strain of *S. marcescens* and four mutants, W_1, O_3, P_2, P_{18}, have been characterized on the basis of their absorption spectra. None of the absorption curves of the mutants resembles the curve of the pigment extract of the HY strain However, the absorption peaks at 538 *mμ* and 370 *mμ* found in the HY strain are also present in the extracts of the red pigment induced by growing the mutant P_2 with W_1 and P_{18} with O_3.

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*THE EFFECTS OF PRE- AND POSTIRRADIATION CENTRIFUGATION ON THE CHROMOSOMES OF TRADESCANTIA AND VICIA**

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According to the most generally accepted theory, ionizing radiations induce chromosome aberrations by first breaking the chromosome threads. Restitution of the majority of these breaks in *Tradescantia*, as calculated by Lea,[1] occurs within 4 minutes. However, those breaks that do not restitute become observable as aberrations either by staying open or by rejoining illegitimately with other chromosomes. Consequently, any disturbance that increases chromosome movement should reduce restitution of broken ends and thereby increase the aberration yield. Previous studies have indeed shown that this is most probably the case. Sax[2] has suggested that centrifugation applied simultaneously with X-radiation causes the broken ends of *Tradescantia* chromosomes to separate, and Conger[3] has indicated that simultaneous sonic and X-ray treatment follows the same principle.

Sax discovered, however, that centrifugation could not increase the aberration yield if it was begun 5 minutes after the irradiation. This result is consistent with the knowledge that nearly all the breaks restitute within 4 minutes. In the seed of *Vicia faba*, in contrast, recent studies[4] have shown that the breaks stay open for 2 hours (i.e., remain capable of fusing with other broken ends). As a result it may be expected that, in *Vicia*, centrifugation could be effective even if started at some considerable time after the irradiation had ceased. Also, on the basis of Steffensen's recent results[5] indicating that in *Tradescantia* there is no restitution within 3 minutes after the irradiation, it may be expected that here, too, postirradiation centrifugation would be effective if begun before chromosome restitution had occurred.

The present study was designed to examine the effects of pre- and postirradiation centrifugation on the chromosomes of *Tradescantia* and *Vicia*. Inflorescences of *Tradescantia paludosa* (clone 3 of Sax) were X-irradiated with a G.E. Maxitron operated at 250 kvp and 30 ma, with 3 mm. of Al filtration. Five days later the microspores were checked for two-hit chromosomal aberrations (ring and dicentrics). Soaked *Vicia* seed were similarly irradiated to produce aberrations observed in the first mitotic root-tip division. All centrifugations were carried out at approximately 2300 g in an International Centrifuge Size 1, Type SB. Preirradiation centrifugation began 15 minutes before irradiation, whereas posttreatment began immediately after irradiation and lasted for 20 minutes. All experiments were performed at room temperature, except experiment 5, in which the inflorescences were kept at 7° C. for 10 minutes immediately after irradiation and then were centrifuged. Three hundred cells were scored for each treatment. The results of these experiments are shown in Table 1.

TABLE 1

EFFECT OF CENTRIFUGATION ON X-RAY-INDUCED CHROMOSOMAL ABERRATIONS

Expt. No.	Dose (r)	Rate (r/Min)	Centrifugation	Percentage Two-Hit Aberrations
Tradescantia:				
1	200	10	None	9.4 ± 1.7
2	200	10	Simultaneously with irradiation	17.0 ± 2.2
3	200	200	None	16.3 ± 2.1
4	200	200	Postirradiation	23.7 ± 2.4
5	200	200	Material kept at 7° C. for 10 minutes postirradiation, then centrifuged	23.0 ± 2.4
6	200	200	Material kept at room temperature 10 minutes postirradiation, then centrifuged	18.0 ± 2.2
7	200	200	Preirradiation	6.4 ± 1.4
8	200	100	None	16.7 ± 2.1
9	200	100	Preirradiation	8.3 ± 1.6
10	200	100	Postirradiation	23.7 ± 2.4
Vicia:				
11	800	200	None	32.3 ± 2.7
12	800	200	Preirradiation	16.0 ± 2.1
13	800	200	Postirradiation	Too numerous to be scored

The effect of postirradiation centrifugation was also tested on aberrations induced under anaerobic conditions. In this case the inflorescences of *Tradescantia* were placed in a gastight Lucite box, and nitrogen was passed over them for one-half hour previous to and during irradiation. The results are given in Table 2.

TABLE 2

EFFECT OF POSTIRRADIATION CENTRIFUGATION ON ABERRATIONS INDUCED IN NITROGEN (*Tradescantia:* 200 r/Min)

Dose (r)	Centrifugation	Percentage Two-Hit Aberrations	Dose (r)	Centrifugation	Percentage Two-Hit Aberrations
175	None	6.0	300	None	13.3
175	20 min.	7.7	300	20 min.	13.7

Discussion.—Table 1 shows that postirradiation centrifugation of *Tradescantia* microspores results in a 20–35 per cent increase in the number of two-hit aberrations (experiment 3 versus experiment 4; experiment 8 versus experiment 10), provided that centrifuging begins immediately after the irradiation has ceased. This may be attributed to the stress and movement of the chromosomes induced by centrifuging. The movement would make restitution less probable and consequently would increase the aberration yield. This result, taken in conjunction with Sax's results that centrifuging after a 5-minute delay does not enhance aberration production, corroborates the hypothesis that the chromosomes do indeed rejoin within 4 minutes. However, if the *Tradescantia* is kept at 7° C. after the irradiation instead of at room temperature, it was found that the ends did not rejoin for as long as 10 minutes, since the centrifuge could be started this late after irradiation and still increase the number of rings and dicentrics (experiment 3 versus experiment 5). This is in agreement with recent results from our laboratory which indicate that low temperatures are able to inhibit chromosome rejoining in *Vicia.*

It may also be noticed that when the radiation is administered at 10 r/min an intensity effect is achieved, and only one-half as many aberrations are formed as when the radiation is given at 200 r/min (experiment 1 versus experiment 3). This intensity effect has been extensively described,[1] the usual explanation being that at low intensities more breaks restitute before the end of the irradiation, leaving fewer breaks to interact and form aberrations. By simultaneous centrifugation and irradiation we were able to increase the yield of aberrations at low intensity (experiment 2) to that at high intensity (experiment 3). Ostensibly, the movement of chromosome ends induced by simultaneous centrifugation prevents restitution and obviates the intensity effect.

In all the experiments performed both in *Tradescantia* and in *Vicia* wherein the material was centrifuged for 15 minutes previous to irradiation, a two- to threefold decrease in aberration yield resulted. This leads us to believe that preirradiation centrifugation, instead of moving ends apart, compresses the chromosomes together at the centrifugal pole. This compression depresses movement of the chromosomes. On this basis, broken ends formed in compressed nuclei are less likely to move apart. This impedance of motion enhances restitution and concurrently decreases the number of aberrations.

In Table 2 it may be observed that, if the material was irradiated in nitrogen, then posttreatment centrifugation did not increase the numbers of aberrations produced. This is consistent with the result that Steffensen (personal communication) describes for chromatid aberrations scored 24 hours after irradiation. It has previously been shown[4, 6] that the breaks induced with anaerobic irradiation in *Vicia* seed rejoin four times faster than those formed in air. Other experiments in our laboratory (unpublished) indicate that the same phenomenon occurs in

Tradescantia and that the anaerobically induced breaks had rejoined before the inflorescences could be transferred from the gas chamber to the centrifuge.

Summary.—It has been observed that in *Tradescantia* microspores and in *Vicia* seed preirradiation centrifugation decreases the number of aberrations as compared to noncentrifuged controls. Postirradiation centrifugation increases aberrations, provided that the centrifuging occurs before the chromosome breaks rejoin.

These phenomena have been explained on the basis of physically induced compression or movement of the chromosomes.

Cells of *Tradescantia* irradiated in nitrogen show no increase of aberration yield caused by centrifugation. This is explained on the basis that rejoining occurs in a shorter time after anaerobic irradiation, as is the case in *Vicia*.

The authors wish to thank Mr. Henry E. Luippold for his technical assistance rendered in the course of these experiments.

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*POSITIVE ZONAL FUNCTIONS ON SPHERES**

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1. We take a fixed real number $\gamma > 0$, and we endow the closed interval $-1 \leq x \leq 1$ with the measure element

$$d\mu(x) = (\int_{-1}^{1} (1 - \xi^2)^{\gamma - 1/2} d\xi)^{-1} (1 - x^2)^{\gamma - 1/2} dx, \tag{1}$$

which, for the special value $\gamma = {}^1/_2(k - 1)$, $k = 2, 3, \ldots$, represents the zonal volume element on the sphere $\xi_1^2 + \ldots + \xi_{k+1}^2 = 1$. We will consider the L_p-classes with the norm

$$f = (\int_{-1}^{1} |f(x)|^p \, d\mu(x))^{1/p},$$

the ordinary class C of continuous functions with the norm max. $|f(x)|$, and the ordinary class V of σ-additive setfunctions $F(A)$ of bounded variation.

We orthogonalize the functions $1, x, x^2, \ldots$, relative to the measure (1) into a sequence of polynomials $P_0(x), P_1(x), P_2(x), \ldots$, but we normalize them thus:

$$P_n(1) = 1. \tag{2}$$

We then have[1]

$$|P_n(x)| \leq 1, \qquad P_n(-1) = (-1)^n, \tag{3}$$

$$\int_{-1}^{1} P_m(x)P_n(x)\, d\mu(x) = \frac{\delta_{mn}}{\rho_m}, \tag{4}$$

$$\rho_n = \frac{\gamma + n}{\gamma}\frac{\Gamma(n + 2\gamma)}{\Gamma(2\gamma)n!} = 0(n^2\gamma), \tag{5}$$

$$\frac{1 - \alpha^2}{(1 - 2\alpha x + \alpha^2)^{\gamma + 1}} = \sum_{n=0}^{\infty} \rho_n \alpha^n P_n(x), \qquad 0 \leq \alpha < 1. \tag{6}$$

The polynomials $C_n^{\gamma}(x) = (-1)^n \binom{-2\gamma}{n} P_n(x)$ satisfy the relations $dC^{\gamma}{}_n(x)/dx = 2\gamma C_{n-1}^{\gamma+1}(x)$, and, from this and (2) and (3), we conclude

$$\frac{1 - P_n(x)}{1 - x} \leq \lim_{x \to 1} \frac{1 - P_n(x)}{1 - x} = \frac{n(n + 2\gamma)}{2\gamma + 1}. \tag{7}$$

2. For functions of C we are introducing a distributive operation which associates with $f(x)$ a symmetric two-variable function $f(x; y)$ by the formula

$$f(x; y) = \int_{-1}^{1} f(xy + (1 - x^2)^{1/2}(1 - y^2)^{1/2}z)\, d\nu(z),$$

$$d\nu(z) = \left(\int_{-1}^{1} (1 - \zeta^2)^{\gamma - 1}\, d\zeta\right)^{-1} (1 - z^2)^{\gamma - 1}\, dz.$$

Obviously, if $a \leq f(x) \leq b$, then also $a \leq f(x; y) \leq b$, but the leading property is

$$P_n(x; y) = P_n(x)P_n(y), \tag{8}$$

which will be decisive for us. The operation can also be extended to functions of L_p and even of V, but we will make no formal statement about this.

3. With $f(x)$ in C or L_p, we can associate the expansion

$$f(x) \sim \sum_{n=0}^{\infty} \rho_n a_n P_n(x), \qquad a_n = \int_{-1}^{1} P_n(y)\, f(y)\, d\mu(y), \tag{9}$$

and more generally with $F(A) \in V$ the expansion

$$F(A) \sim \sum \rho_n a_n \int_A P_n(x)\, d\mu(x), \qquad a_n = \int_{-1}^{1} P_n(y)\, dF(y), \tag{10}$$

and the classical theorem on the moment problem implies that the series determines its function uniquely. Next, if the series (9) is finite or absolutely convergent, then by (8) we have

$$f(x; y) \sim \sum \rho_n a_n P_n(x)P_n(y), \tag{11}$$

and this implies, in particular,

$$\int_{-1}^{1} f(x; y)d\mu(x) = \int_{-1}^{1} f(x; y)d\mu(y) = \int_{-1}^{1} f(x)\, d\mu(x) = a_0.$$

Also, if we take a second function

$$g(x) \sim \sum \rho_n b_n P_n(x), \tag{12}$$

then we have

$$(\varphi(x) \equiv) \int_{-1}^{1} f(y)g(y; x)\, d\mu(y) = \int_{-1}^{1} f(x; y)g(y)\, d\mu(y)$$

$$= \sum \rho_n a_n b_n P_n(x),$$

and

$$\varphi(x;\, y) \,=\, \int_{-1}^{1} f(x;\, z) g(z;\, y)\, d\mu(z) \,=\, \sum \rho_n a_n b_n P_n(x) P_n(y).$$

4. We take a polynomial $k(x)$ for which

$$k(x) \,\geq\, 0, \qquad \int_{-1}^{1} k(x;\, y)\, d\mu(y) \,=\, \int_{-1}^{1} k(y)\, d\mu(y) \,=\, 1,$$

and for any $f(x) \,\epsilon\, C$ or $\epsilon\, L^p$ we form

$$f_0(x) \,=\, \int_{-1}^{1} f(y) k(y;\, x)\, d\mu(y).$$

We claim that we have

$$\|f_0\| \,\leq\, \|f\| \tag{13}$$

in either norm, and that, for $F \,\epsilon\, V$, for

$$f_0(x) \,=\, \int_{-1}^{1} k(y;\, x)\, dF(y)$$

we have

$$\int_{-1}^{1} |f_0(x)|\, d\mu(x) \,\leq\, \int_{-1}^{1} |dF(y)|. \tag{14}$$

In fact, for $f \,\epsilon\, C$ we have

$$|f_0(x)| \,\leq\, \|f\| \int_{-1}^{1} k(y;\, x)\, d\mu(y) \,=\, \|f\|,$$

whence the conclusion, and for $F \,\epsilon\, V$ we obtain

$$|f_0(x)| \,\leq\, \int_{-1}^{1} k(y;\, x)\, |dF(y)|,$$

which implies (14) after integration with respect to x. Finally, for $f \epsilon\, L_p$, $p > 1$, on putting $k(y;\, x) \,=\, k^{1/p \,+\, 1/q}$ we obtain from Holder's inequality

$$|f_0(x)|^p \,\leq\, \int_{-1}^{1} |f(y)|^p\, k(y;\, x)\, d\mu(y) \int_{-1}^{1} k(y;\, x) d\mu(y) \,=\, \int_{-1}^{1} |f(y)|^p\, k(y;\, x) d\mu(y),$$

and now we integrate with respect to x.

5. We now take a fixed sequence of functions

$$k^r(x) \,\sim\, \sum \rho_n\, \lambda_n{}^r P_n(x), \qquad r \,=\, 1, 2, 3, \ldots,$$

with the following properties: (i) Each $k^r(x)$ is a polynomial, so that

$$\lambda_n{}^r \,=\, 0 \text{ for } n \,\geq\, N(r); \tag{15}$$

(ii) we have $k^r(x) \,\geq\, 0$ and $\int_{-1}^{1} k^r(x)\, d\mu(x) \,=\, 1$, so that, in particular,

$$\lambda_0{}^r \,=\, 1, \; |\lambda_n{}^r| \,\leq\, 1; \tag{16}$$

and (iii) for any $\epsilon > 0$ we have $\lim_{r \,\to\, \infty} \int_{1 \,-\, \epsilon}^{1} k^r(x)\, d\mu(x) \,=\, 1$, so that by (2) we have

$$\lim_{r \,\to\, \infty} \lambda_n{}^r \,=\, 1, \qquad n \,=\, 0, 1, 2, \ldots. \tag{17}$$

THEOREM 1.[2] *If $f \,\epsilon\, C$ or ϵL_p, then the approximating sequence*

$$f^r(x) \,=\, \int_{-1}^{1} f(y) k^r(y;\, x)\, d\mu(y) \,=\, \sum \rho_n\, \lambda_n{}^r a_n P_n(x) \tag{18}$$

converges in norm,

$$\lim_{r \,\to\, \infty} \|f \,-\, f^r\| \,=\, 0, \tag{19}$$

and for $F \in V$ the sequence

$$f^r(x) = \int_{-1}^{1} k^r(y; x)\, dF(x) = \sum \rho_n \lambda_n{}^r a_n P_n(x)$$

is such that for g in C we have

$$\int_{-1}^{1} g(x) f^r(x)\, d\mu(x) \to \int_{-1}^{1} g(x)\, dF(x). \tag{20}$$

In fact, it follows from (17) that (19) and (20) hold if $f(x)$ or $g(x)$ is a polynomial $f_\epsilon(x)$ or $g_\epsilon(x)$. For a general $f(x)$ we approximate to it by a polynomial $f_\epsilon(x)$ in norm, $\|f - f_\epsilon\| \leq \epsilon$, and (13) then implies $\|f^r - f_\epsilon{}^r\| \leq \epsilon$, which proves (19). Also, (14) implies $\left|\int_{-1}^{1} (g - g_\epsilon) f^r\, d\mu\right| \leq \epsilon \int_{-1}^{1} |dF(x)|$, which proves (20).

6. We call a sequence of numbers $c_0, c_1, c_2, \ldots$ *positive-definite* if, for any M, whenever

$$(f(x) \equiv) \sum_0^M \rho_n a_n P_n(x) \geq 0, \tag{21}$$

then also

$$(\varphi(x) \equiv) \sum_0^M \rho_n a_n c_n P_n(x) \geq 0. \tag{22}$$

THEOREM 2. *A sequence $\{c_n\}$ is positive-definite if and only there is an $H(A)$ in V^+ such that*

$$c_n = \int_{-1}^{1} P_n(x)\, dH(x). \tag{23}$$

In fact, if (23) holds, then

$$\varphi(x) = \int_{-1}^{1} f(x; y)\, dH(y) \geq 0.$$

In the converse case we assume first that $c_n = 0$ for $n \geq N$. If we then put $g(x) = \sum \rho_n c_n P_n(x)$, then (22) says that we have $\int_{-1}^{1} g(x) f(x)\, dx \geq 0$ for all polynomials $f(x) \geq 0$. But this implies $g(x) \geq 0$, as claimed. For a general sequence $\{c_n\}$ we introduce the approximating functions (18), and, since (21) implies $f^r(x) \geq 0$, assumption (22) implies

$$\sum_0^M \rho_n a_n c_n \lambda_n{}^r P_n(x) \geq 0.$$

Thus $c_n{}^r \equiv c_n \lambda_n{}^r$ is a positive definite sequence for each r. But $c_n{}^r = 0$ for $n \geq N(r)$, and by what we have just proved we have, therefore,

$$c_n{}^r = \int_{-1}^{1} P_n(x)\, dH^r(x)$$

for an element $H^r(A)$ in V^+. Since $\int_{-1}^{1} dH^r(x) = c_0$ is independent of r, a subsequence $H^r{}_s(A)$ is weakly convergent to an element $H(A)$ in V^+. By (17) the Fourier coefficients of $H(A)$ are c_n.—*Q.E.D.*

7. We say that a sequence of functions $\{c_n(t)\}$, $0 \leq t < \infty$, $n = 0, 1, 2, \ldots$ is a *homogeneous (stochastic) process* if (i) for each t, $\{c_n(t)\}$ is a positive-definite sequence;

(ii) $c_n(u+v) = c_n(u)c_n(v)$; (iii) $c_0(t) = 1$; and (iv) $c_n(0) = 1$ and $c_n(t)$ is continuous.

THEOREM 3.[3] *A sequence $\{c_n(t)\}$ is a homogeneous process if and only if there is in the half-open interval $-1 \leq x < 1$ a σ-additive setfunction $G(A) \geq 0$ for which*

$$\int_{-1}^{1-0} dG(x) < \infty \tag{24}$$

and a constant $\sigma \geq 0$ such that

$$c_n(t) = \exp\left\{-t\,\sigma n(n+2\gamma) - t\int_{-1}^{1-0} \frac{1-P_n(y)}{1-y}\,dG(y)\right\}. \tag{25}$$

Proof: Our leading property (8) means that the sequence $c_n = P_n(y)$ is positive-definite for each y in $-1 \leq y \leq 1$. By simple closure properties of such sequences this also holds true for the sequence $\int_{-1}^{1} P_n(y)\,dH(y)$ for any $H(A)$ in V^+ and then also for

$$\left\{\int_{-1}^{1} P_n(y)\,dH(y)\right\}^r$$

for any $r = 0, 1, \ldots$. If we multiply by $t^r/r!\exp\{-t\int_{-1}^{1} dH(y)\}$ and sum over r, we obtain the conclusion that

$$\exp\left\{-t\int_{-1}^{1}(1-P_n(y))\,dH(y)\right\}$$

is positive-definite if $H(A) \geq 0$ and $\int_{-1}^{1} dH(x) < \infty$. But now we take any sequence of such elements $H^r(A)$ which is such that

$$\lim_{r\to\infty}\int_{-1}^{1}(1-P_n(y))\,dH^r(y)$$

exists for every n. Due to (3) and (7), it follows as in the classical "Euclidean" case, that every sequence (25) with (24) can be obtained in this way. Thus any such expression (25) has property (i) of a homogeneous process, and the other properties are trivially verified from the expression.

Conversely, properties (ii), (iii), (iv), of a sequence imply that we can put

$$c_n(t) = \exp(-t\gamma_n),$$

where

$$\gamma_n = \lim_{t\downarrow 0}\frac{1-c_n(t)}{t},$$

the limit existing. Now, by properties (i) and (iii), Theorem 2 implies that we have

$$\gamma_n = \lim_{t\downarrow 0}\int_{-1}^{1}(1-P_n(y))\,d_x\frac{H(y;t)}{t},$$

where $H(A;t) \in V^+$ for each t. Now, again by (3) and (7), such a limit must be of the form (25).—*Q.E.D.*

THEOREM 4. *The constant σ and the set function $G(A)$ in (25) are uniquely determined.*

Proof: By (3) and (7) we have

$$\int_{-1}^{1-0}\frac{1-P_n(y)}{1-y}\,dG(y) = o(n^2),$$

which proves the uniqueness of σ. And, by the solution of the moment problem the uniqueness of $G(A)$ follows then from the fact that any polynomial in y is a linear combination with constant coefficients, of the factors $(1 - P_n(y))/(1 - y)$.

8. Perhaps the most peculiar conclusion from the first part of Theorem 3 is the fact that the function

$$\sum_{n=0} \rho_n e^{-tn(n+2\gamma)} P_n(x) P_n(y) \tag{26}$$

is nonnegative for $-1 \leq x \leq 1$, $-1 \leq y \leq 1$. So is also, of course, every function presentable in the form

$$\sum_{n=0} \rho_n \exp\left\{-t\sigma n(n+2\gamma) - t \int_{-1}^{1-0} \frac{1 - P_n(z)}{1 - z}\, dG(z)\right\} P_n(x) P_n(y),$$

and it follows from a general theorem[4] that this includes, in particular, all functions of the form

$$\sum \rho_n \exp\{-t\,\phi\,(n(n+2\gamma))\} P_n(x) P_n(y), \tag{27}$$

where

$$\phi(\xi) = \sigma\xi + \int_{+0}^{\infty} \frac{1 - e^{-\rho\xi}}{\rho}\, d\gamma(\rho),$$

$$\sigma \geq 0,\ d\gamma(\rho) \geq 0, \qquad \int_{+0}^{\infty} d\gamma(\rho) < \infty.$$

Special cases of this are

$$\sum \rho_n e^{-t(n(n+2\gamma))} P_n(x) P_n(y) \tag{28}$$

and

$$\sum \rho_n e^{-tn} P_n(x) P_n(y), \tag{29}$$

and the positiveness of the last series follows also from (6) directly.

Now (26) is a solution of the diffusion equation

$$\Delta_x f = (x^2 - 1) \frac{\partial^2 f}{\partial x^2} + (2\gamma + 1)\, x \frac{\partial f}{\partial x} = -\frac{\partial f}{\partial t}$$

in $(-1, 1)$, and (28) of the equation

$$\Delta^{1/2} f = -\frac{\partial f}{\partial t}.$$

But (29) is a solution of the somewhat more complicated equation

$$\{(\Delta + 4\gamma^2 \cdot 1)^{1/2} - 2\gamma \cdot 1\}_x f = -\frac{\partial f}{\partial t},$$

and in this sense function (28) pertains to a "simpler" problem than does (29), appearances notwithstanding.

* This work was begun in summer, 1954, at Cornell University, while the author was there in connection with a project sponsored by the Office of Naval Research, under the directorship of J. Wolfowitz.

[1] For formulas cited without proof see W. Magnus and F. Oberhettinger, *Formulas and Theorems for the Special Functions of Mathematical Physics* (Chelsea, 1949).

[2] This theorem imitates known facts for Fourier expansions on compact group and homogeneous spaces. For summability at points see G. Szegö, *Orthogonal Polynomials* (1939) and references there listed to publications by Kogbetliantz, Gronwall, Fejér, and others.

[3] To Theorems 3 and 4 compare S. Bochner, "Closure Classes Originating in the Theory of Probability," these PROCEEDINGS, **39**, 1082–1088, 1953.

[4] See S. Bochner, "Diffusion Equation and Stochastic Processes," these PROCEEDINGS, **35**, 368–370, 1949.

KÄHLERIAN COSET SPACES OF SEMISIMPLE LIE GROUPS

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Our main purpose in this note is to determine the coset spaces of semisimple Lie groups which admit a complex analytic Kählerian structure invariant under the group; we shall also obtain some information on coset spaces with an invariant symplectic structure. In the compact case, all the manifolds thus obtained are algebraic and admit a complex analytic cellular decomposition; in the noncompact case, they are complex analytically fibered, with compact Kählerian fibers, over Hermitian symmetric spaces. As an application, we see that a bounded domain in the space of several complex variables which has a transitive *semisimple* group of complex analytic homeomorphisms is symmetric in É. Cartan's sense,[1] thus giving a *partial* answer to a well-known question raised by that author. Only brief indications of proofs are given; the full details will appear elsewhere.

1. *Notations and Definitions.*—G denotes a connected Lie group, which, except in section 1, *is always supposed to be semisimple;* U is a closed subgroup of G; and G/U is the space of left cosets of G modulo U, on which G acts by the left translations. We always assume G to be effective on G/U, i.e., U contains no subgroup $\neq \{e\}$ invariant in G. Lie algebras are denoted by German letters and, unless otherwise stated, are taken over the real numbers; the Lie algebra of a group G, $U, \ldots$ is of course denoted by the corresponding German letter.

A complex analytic manifold is Kählerian if it is endowed with a Hermitian metric whose imaginary part Ω, the so-called associated form to the metric, has exterior differential zero; in any case it is an exterior form of degree two and maximal rank everywhere. An even-dimensional manifold carrying a form Ω with the last-named properties is called *symplectic;* it is always orientable; clearly, Kählerian implies symplectic.

A coset space is homogeneous complex (resp. homogeneous Kählerian, resp. homogeneous symplectic) if it carries a complex analytic structure (resp. Kählerian structure, resp. a form of degree two and maximal rank everywhere) invariant under the group. For a compact connected group the usual averaging process shows that symplectic implies homogeneous symplectic and that "homogeneous complex and Kählerian" implies "homogeneous Kählerian."

$H^i(X)$ (resp. $H^i(X, Z)$) is the ith cohomology group of the manifold X with real coefficients (resp. integers).

2. *A Necessary Condition.*—A Lie algebra is reductive[2] if its adjoint representation is fully reducible or, equivalently,[3] if it is the direct product of its center by a semisimple ideal; a Lie subalgebra $\mathfrak{b}$ of a Lie algebra $\mathfrak{a}$ is reductive in $\mathfrak{a}$ if the restriction to $\mathfrak{b}$ of the adjoint representation of $\mathfrak{a}$ is fully reducible in $\mathfrak{a}$.

PROPOSITION 1. *Let G/U be homogeneous symplectic, U be connected. Assume $\mathfrak{u}$ to be reductive in $\mathfrak{g}$, and let $\mathfrak{c}$ be its center. Then $\mathfrak{u}$ is the centralizer of $\mathfrak{c}$ in $\mathfrak{g}$.*

The proof makes mainly use of cohomology of Lie algebras and is based on the three following facts: (*a*) a semisimple Lie algebra has vanishing first and second cohomology groups; (*b*) the centralizer of $\mathfrak{c}$ in $\mathfrak{g}$ is reductive in $\mathfrak{g}$; (*c*) there exists an element h in the second relative cohomology group $H^2(\mathfrak{g}, \mathfrak{u})$ such that $h^m \neq 0$ ($2m = \dim G/U$).

Remarks: (1) It follows from Proposition 1 that $\mathfrak{u}$ contains a Cartan subalgebra of $\mathfrak{g}$. (2) Proposition 1 applies when G/U is homogeneous Kählerian, because in that case $\mathfrak{u}$ is readily seen to be reductive in $\mathfrak{g}$, even when U is not compact. It also applies for G compact, G/U symplectic. (3) For a particular case of Proposition 1, see A. Lichnerowicz, *Compt. Rend. Acad. Sci.* (*Paris*) **237,** 695–697 (1953).

THEOREM 1. *Let either G/U be homogeneous Kahlerian, or G be compact and G/U be symplectic. Then U is compact, connected, and equal to the centralizer of a torus of G. Moreover, G has center reduced to $\{e\}$,[4] and G/U is simply connected.*

For U connected, this follows essentially from Proposition 1 and remark 1. In the general case, one considers the covering G/U_0 (U_0 connected component of the identity in U), on which the given structure on G/U induces a similar structure, invariant under G, and also under U/U_0 operating by right translations; the main point is that U/U_0 operates faithfully on the identity component of the center of U_0, and that follows from the lemma: If the centralizer of a torus in a connected semisimple Lie group has a compact identity component, then it is equal to it.

COROLLARY. *Let $G = G_1 \times \ldots \times G_k$ be a decomposition of G into a product of simple groups. Then $U = U_1 \times \ldots \times U_k$ where $U_i \subset G_i$ and is the centralizer of a torus in G_i; hence G/U is isomorphic*[5] *to the product of the spaces G_i/U_i.*

3. *Complex Semisimple Lie Algebras.*—We recall here a few known facts and fix some notations. Let $\mathfrak{g}$ be a compact semisimple Lie algebra of rank l, dimension $n = l + 2m$; let $\mathfrak{g}^c$ be the complexification of $\mathfrak{g}$ and $\mathfrak{h}^c$ be a Cartan subalgebra of $\mathfrak{g}^c$ such that $\mathfrak{h} = \mathfrak{h}^c \cap \mathfrak{g}$ has (real) dimension l. We denote by $\pm 2\pi i a_j$ ($1 \leqq j \leqq m$) the roots of $\mathfrak{g}^c$ with respect to $\mathfrak{h}^c$; the a_j's are therefore real-valued on $\mathfrak{h}$, and, moreover, we assume them to be positive with respect to some total ordering of the space $\mathfrak{h}^*$ dual to $\mathfrak{h}$, chosen once for all, the fundamental roots being $2\pi i a_k$ ($1 \leqq k \leqq l$). The scalar product induced on $\mathfrak{h}$ or $\mathfrak{h}^*$ by the Killing form is written $(\ ,\ )$, and W is the Weyl chamber (in $\mathfrak{h}$ or $\mathfrak{h}^*$) defined by $(a_k, y) \geqq 0$ ($1 \leqq k \leqq l$).

$e_{\epsilon j}$ ($\epsilon = \pm 1, j = 1, \ldots, m$) is an element of $\mathfrak{g}^c$ satisfying

$$[h, e_{\epsilon j}] = \epsilon 2\pi i a_j(h) \qquad (h \in \mathfrak{h}),$$

chosen in the usual way, and $\mathfrak{g}$ is spanned over the reals by $\mathfrak{h}$, by $e_j + e_{-j}$ and $i(e_j - e_{-j})$ ($1 \leqq j \leqq m$).

Let G^c (resp. G) be the group with center reduced to $\{e\}$ and Lie algebra $\mathfrak{g}^c$ (resp. $\mathfrak{g}$). We denote by L the closed solvable subgroup of G^c generated by $\mathfrak{h}^c$ and the e_j's ($1 \leqq j \leqq m$) and by L_b ($b \in W$) the subgroup generated by $\mathfrak{h}^c$, the e_j's ($1 \leqq j \leqq m$), and the e_{-k} for which $(a_k, b) = 0$ (it is readily seen that these elements form the Lie algebra of a closed subgroup).

$\mathfrak{h}^*$ may be identified in a well-known way to $H^1(T)$, where T is a maximal torus of G with Lie algebra $\mathfrak{h}$; in this identification $H^1(T, Z)$ becomes the set of $h \in \mathfrak{h}^*$ for which $2(a_k, h)(a_k, a_k)^{-1}$ ($1 \leqq k \leqq l$) is an integer.

4. *The Compact Case.*—If G is compact semisimple, then G/U has the first Betti number zero; conversely, any compact coset space of a Lie group with vanishing first Betti number is a quotient of a compact group[6] which, as is easily seen, may be assumed to be semisimple. Hence the results of sections 2 and 4 give *all algebraic homogeneous manifolds with first Betti number zero.*

THEOREM 2. *Let G be compact semisimple and U be the centralizer of a torus. Then G/U is homogeneous Kählerian and algebraic.*[7]

There exists clearly $b \in W$ such that $\mathfrak{u}$ is the centralizer of b; one proves that $U = G \cap L_b$ (notations of sec. 3), whence a natural homeomorphism of G/U onto G^c/L_b commuting with G and the homogeneous complex structure. The invariant Kählerian metric is then constructed by means of Maurer-Cartan forms. We sketch here the proof for G/T (T maximal torus), i.e., $L_b = L$; the general case is analogous.

Let us denote by ω^{*j} the left-invariant Maurer-Cartan forms on G^c which induce on $\mathfrak{g}^c$ the base dual to (e_{*j}) and are orthogonal to $\mathfrak{h}^c$; using the Maurer-Cartan equations and well-known properties of constants of structure, one shows that

$$\Omega = i \sum_{j=1}^{j=m} c_j \omega^j \wedge \omega^{-j}$$

is closed if and only if

$$c_p + c_q = c_r \qquad \text{whenever} \qquad a_p + a_q = a_r.$$

Ω is therefore determined by the c_k's ($1 \leqq k \leqq l$), which are arbitrary; its restriction on G is left-invariant under G, right-invariant under T, and represents a form on G/T, which is of type (1, 1) because ω^{-j} corresponds to $\bar{\omega}^j$ in the complex structure constructed above. For real c_j's it is real-valued, and its (real) cohomology class may be shown to be the image by transgression of the element $h \in H^1(T)$, for which $(a_k, h) = c_k$ ($1 \leqq k \leqq l$). If h belongs to the interior of W, all the c_j's are > 0, and

$$ds^2 = \sum_{j=1}^{j=m} c_j \omega^j \cdot \bar{\omega}^j \qquad \text{(usual product)}$$

is a Kählerian metric on G/T. If, moreover, $h \in H^1(T, Z)$, its image by transgression is an integral class, the corresponding metric is a Hodge metric, and G/T is algebraic by a result of Kodaira.[8]

Remark: This theorem can be proved in other ways; for instance, one can construct projective imbeddings with the help of linear representations, as was noticed by J. Tits and, independently, by A. Weil and the author (yet unpublished); also, M. Goto proved that G/U is rational algebraic (to appear in *Am. J. Math*); finally,

the existence of the homogeneous complex structure is part of a result of H. C. Wang (*Am. J. Math.*, **76,** 1–32, 1954). The above method has been sketched, however, because it is also used in section 6.

5. *Complex Analytic Cellular Decompositions.*—A complex compact manifold M has a complex analytic cellular decomposition if it admits a partition into a finite number of (complex) submanifolds M_i, the "open cells," each isomorphic to some complex affine space and having a set-theoretical boundary made up of open cells with strictly smaller dimensions; the closures of the M_i's define, then, a cellular decomposition, whose cells have *even* dimensions; consequently, they are all cycles, and they form a basis for the integral homology groups of M, which thus have no torsion and vanish in odd dimensions. As is well known, C. Ehresmann[9] proved the existence of such decompositions for certain classical spaces, like the complex Grassmann manifolds or the nondegenerate complex quadrics; his method is geometric and uses mainly the Schubert systems, but this result can also be given a group-theoretical proof, valid for all spaces considered in section 4.

THEOREM 3. *Let G be compact and G/U be homogeneous Kählerian. Then G/U admits a complex analytic cellular decomposition by "open cells" which are birationally and biregularly equivalent to complex affine spaces; in particular, its integral homology groups have no torsion.*[10]

If we represent G/U in the form G^c/L_b as above, the open cells are just the orbits of L; to prove it, one uses notably a recent (unpublished) result of Harish-Chandra (first checked by Bruhat for the classical groups, as announced in *Compt. rend. Acad. sci.* (Paris), **238,** 437, 1954), to the effect that the double cosets LgL of L in G^c are finite in number.

6. *The Noncompact Case.*—A Hermitian manifold is *Hermitian symmetric* if every point is an isolated fixed point of an involutive automorphism of the Hermitian structure; it is then always homogeneous Kählerian.[11] The quotient G/K of a simple noncompact group with center reduced to $\{e\}$ by a maximal compact subgroup carries a Hermitian symmetric structure invariant under G if and only if K has a nondiscrete center.[11]

PROPOSITION 2. *Let G be simple noncompact, with center reduced to $\{e\}$, K a maximal compact subgroup of G, and U a subgroup of K which is the centralizer in G of a torus. Then G/U is homogeneous complex and homogeneous symplectic.*[12] *It is homogeneous Kählerian if and only if G/K is Hermitian symmetric; in that case, the fibering of G/U by K/U over G/K is complex analytic.*

Let G_c be a maximal compact subgroup of the complexification G^c of G, containing K; the group U is also centralizer of a torus in G_c, and (Theorem 2) G_c/U is homogeneous Kählerian; it is easily shown that G/U may be identified with an open submanifold of G_c/U, whence the homogeneous complex structure. The second part of the theorem is obtained by detailed analysis of invariant differential forms. From that and from the corollary to Theorem 1, one gets Theorem 4.

THEOREM 4. *The homogeneous Kählerian coset spaces of semisimple Lie groups are all simply connected. They are exactly the products of the Kählerian homogeneous spaces G_i/U_i with G_i simple, U_i centralizer of a torus, and where either G_i is compact or G_i has a maximal compact subgroup with nondiscrete center. Any coset space G/U of this type has a complex analytic fibering with fiber K/U (K maximal compact) over a Hermitian symmetric space.*

A bounded domain in C^n possesses a Kählerian metric invariant under all complex analytic homeomorphisms (the Bergmann metric); hence, if it is homogeneous, it is automatically homogeneous Kählerian. It is said to be *symmetric*[1] if every point is an isolated fixed point of an involutive complex analytic homeomorphism; this implies Kählerian homogeneity. É. Cartan[1] has asked whether every bounded homogeneous domain in C^n is symmetric and has checked that it is indeed the case for $n = 1, 2, 3$. Since a domain does not contain a connected compact complex analytic submanifold with more than one point, we deduce from Theorem 4:

THEOREM 5. *A bounded domain in C^n which admits a transitive semisimple group of complex analytic homeomorphisms is symmetric.*[13]

[1] É. Cartan, *Abhandl. Math. Sem. Hamburg*, **11**, 116–162, 1935.

[2] J. L. Koszul, *Bull. Soc. Math. France*, **78**, 65–127, 1950.

[3] *Ibid.*, p. 87.

[4] Recall that G is effective on G/U by assumption.

[5] As coset space only.

[6] D. Montgomery, *Proc: Am. Math. Soc.*, **1**, 467–469, 1950.

[7] I.e., is complex analytically homeomorphic to a complex submanifold of some complex projective space, imbedded without singularities.

[8] K. Kodaira, these PROCEEDINGS, **40**, 313–316, 1954.

[9] C. Ehresmann, *Ann. Math.*, **35**, 396–443, 1934.

[10] This applies, e.g., to G/T (T maximal torus), as had been partly checked by the author (*Ann. Math.*, **57**, 115–207, 1953, sec. 29). The absence of torsion on these spaces has also been proved by R. Bott, these PROCEEDINGS, **40**, 586–588(1954).

[11] A. Borel and A. Lichnerowicz, *Compt. rend. Acad. sci.* (Paris), **234**, 2332–2334, 1952.

[12] In fact, it always carries an invariant *indefinite* Kählerian metric.

[13] Theorem 5 has also been obtained independently by J. L. Koszul (yet unpublished). For the homogeneous spaces of theorem 2, see also J. Tits, *Compt. Rend. Acad. Sci. (Paris)*, **239**, 466–468(1954).

THE GREEN'S AND NEUMANN'S PROBLEMS FOR DIFFERENTIAL FORMS ON RIEMANNIAN MANIFOLDS

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1. *Definition of the Green's and Neumann's Spaces.*—The classical Green's and Neumann's problems for a function on a subdomain of Euclidean n-space are the problems of finding a harmonic function with prescribed values on the boundary for the function and its normal derivative respectively. Our purpose here is to state a generalization of these problems to differential forms on an arbitrary Riemannian manifold; detailed proofs will appear elsewhere.

By "differentiable" we shall always mean "differentiable of class C^∞," and by "manifold" we shall mean "orientable differentiable manifold." All Riemannian structures are therefore of class C^∞.

On a Riemannian manifold of dimension n we have, in addition to the operator d of exterior differentiation, its formal adjoint $\delta = (-1)^{np+n+1} * d *$ where $*$ denotes the usual duality operator carrying a differential form of degree p (p-form) into one

of complementary degree $n - p$. We also have the scalar product (φ, ψ) of the forms φ, ψ, and we write $\|\varphi\| = \sqrt{(\varphi, \varphi)}$. If φ is not homogeneous, say $\varphi = \sum\varphi^p$, where φ^p is of degree p, the norm is $\|\varphi\|^2 = \sqrt{\sum\|\varphi^p\|}$.[2] We denote by H the Hilbert space of norm-finite differential forms on R; then $H = \sum H^p$, where H^p is the subspace of norm-finite r-forms.

We say that a form φ is in the domain of the operator d, and we write $\varphi \epsilon d$ if and only if there exists a sequence $\{\varphi_\mu\}$ of differentiable forms φ_μ, $\|\varphi_\mu\| + \|d\varphi_\mu\| < \infty$ such that $\varphi\mu$ converges to φ and $d\varphi\mu$ converges (both in the sense of the norm). We denote the limit of $d\varphi\mu$ by $d\varphi$. It is readily seen that $d\varphi$ is well defined. If there exists such a sequence where each $\varphi\mu$ has compact support, we say that φ is in the domain of the operator d_c, we write $\varphi \epsilon d_c$, and we denote the limit of $d\varphi\mu$ by $d_c\varphi$, and $d_c\varphi$ is again seen to be well defined. In a similar fashion we define δ and δ_c. It is possible to show that d is the Hilbert space adjoint of δ_c, δ the Hilbert space adjoint of d_c.[1] Therefore, the domain of the operator d_c consists of those forms which, in a generalized sense, have a vanishing tangential component at the (ideal) boundary of the manifold R, while the domain of the operator δ_c consists of those forms with vanishing normal component at the "boundary." It is obvious that these two domains are dual in the sense of the operator*.

Now let F denote the subspace of H composed of forms satisfying $d\varphi = \delta\varphi = 0$, and let F_t, F_n be the intersection of F with the domains of d_c, δ_c, respectively. Then F_t is the space of harmonic forms with vanishing tangential component, F_n the space of harmonic forms with vanishing normal component. Denote by $[d\Phi]$ the closure, under the norm, of the space $\{d\varphi \mid \varphi \epsilon d\}$, and let $[\delta\Phi]$, $[\delta\Phi]$ be defined in a similar fashion. We have the following three formulas of orthogonal decomposition:

$$(1)\ H = [d_c\Phi] + [\delta_c\Phi] + F;$$
$$(2)\ H = [d_c\Phi] + [\delta\Phi] + F_t;$$
$$(3)\ H = [d\Phi] + [\delta_c\Phi] + F_n.$$

Corresponding to these decompositions, we introduce the following three Laplace operators:

$$(1)\ \Delta_N = \delta_c d + d_c\delta;$$
$$(2)\ \Delta_L = \delta d_c + d_c\delta;$$
$$(3)\ \Delta_M = \delta_c d + d\delta_c.$$

We denote the domains of these operators by N, L, and M, respectively. It is clear that N consists of forms φ such that the normal component of $d\varphi$ and the tangential component of $\delta\varphi$ vanish at the boundary, that L consists of forms φ with the tangential components of φ and $\delta\varphi$ vanishing, while M consists of forms φ with the normal components of φ and $d\varphi$ vanishing at the boundary. The operator Δ_N is investigated in the paper by Gaffney,[1] where it is shown, in particular, that the domain N of Δ_N coincided with the "Neumann's space" first introduced by Spencer.[2, 3]

Since every function may trivially be regarded as belonging to the domain of δ_c, and therefore as having a vanishing normal component at the boundary, we see that M^0 coincides with N^0, where N^0 is the Neumann's space of functions with

vanishing normal derivative at the boundary. On the other hand, L^0 is obviously the Green's space of functions which vanish at the boundary; therefore, $M = \sum M^r$ and $L = \sum L^r$ may be regarded as being, respectively, the Neumann's and Green's space of differential forms, these spaces being dual under the operator $*$.

2. *The Green's and Neumann's Operators.*—In his paper[1] Gaffney proves that Δ_N is a self-adjoint operator, and the same method shows that Δ_L and Δ_M are self-adjoint. Let I stand for the identity operator, and let s denote an arbitrary, but fixed, positive number. From the general theory of Hilbert space[4] the operators $\Delta_L + sI$, $\Delta_M + sI$, and $\Delta_N + sI$ have inverses which we denote by L_s, M_s, and N_s, respectively. These inverses are everywhere defined and are easily seen to be bounded by $1/s$. Corresponding to the three formulas of orthogonal decomposition listed in section 1, we have decompositions of N_s, L_s, and M_s, namely,

$$(1') \quad N_s = N_s^{(1)} + N_s^{(2)} + (1/s)F,$$
$$(2') \quad L_s = L_s^{(1)} + L_s^{(2)} + (1/s)F_t,$$
$$(3') \quad M_s = M_s^{(1)} + M_s^{(2)} + (1/s)F_n,$$

where F, F_t, and F_n are the operators of orthogonal projection onto the spaces F, F_t, and F_n, respectively, and where, given $\varphi \in H$, $N_s^{(1)}(\varphi) \in [d_c\Phi]$, $N_s^{(2)}(\varphi) \in [\delta_c\Phi]$, $L_s^{(1)}(\varphi) \in [d_c\Phi]$, $L_s^{(2)}(\varphi) \in [\delta\Phi]$, $M_s^{(1)}(\varphi) \in [d\Phi]$, $M_s^{(2)}(\varphi) \in [\delta_c\Phi]$.

THEOREM 1. *The operators d and δ_c commute with M_s (that is, $dM_s\varphi = M_s d\varphi$ if $\varphi \in d$ and $\delta_c M_s\varphi = M_s\delta_c\varphi$ if $\varphi \in \delta_c$), and the space of eigenforms of the operator M_s belonging to the eigenvalue $1/s$ coincides with F_n, that is*

$$F_n = \{\varphi \mid \varphi \in H,\ \Delta_M M_s(\varphi) = \varphi - sM_s\varphi = 0\}. \tag{2.1}$$

Moreover, $M_s^{(2)}d = dM_s^{(1)}$. If $\varphi \in d$, then $M_s^{(2)}\varphi = 0$ if and only if $d\varphi = 0$; if $\varphi \in \delta_c$, $M_s^{(1)}\varphi = 0$ if and only if $\delta_c = 0$. Dual statements apply to the operator L_s, and we have the formula

$$N_s = L_s^{(1)} + M_s^{(2)} + (1/s)F. \tag{2.2}$$

That is, $N_s^{(1)} = L_s^{(1)}$, $N_s^{(2)} = M_s^{(2)}$.

We remark that the operators L_s and M_s, like N_s, can be defined by Dirichlet's principle.[3] In fact, given $\beta \in H$, $M_s\beta$ is the form which minimizes the expression $(d\varphi, d\varphi) + (\delta_c\varphi, \delta_c\varphi) + s(\varphi - (1/s)\beta, \varphi - (1/s)\beta)$.

3. *Semigroups Corresponding to the Green's and Neumann's Operators.*—The toerators Δ_L, Δ_M, and Δ_N are self-adjoint positive operators on H, and the spectral heporem asserts the existence of corresponding increasing families of projections which we denote, respectively, by $\{E_L(\lambda)\}$, $\{E_M(\lambda)\}$, and $\{E_N(\lambda)\}$ such that

$$\Delta_L = \int \lambda\, dE_L(\lambda), \qquad \Delta_M = \int \lambda\, dE_M(\lambda), \qquad \Delta_N = \int \lambda\, dE_N(\lambda). \tag{3.1}$$

We form the corresponding semigroups

$$U_t = \int e^{-\lambda t}\, dE_L(\lambda), \qquad V_t = \int e^{-\lambda t}\, dE_M(\lambda), \qquad W_t = \int e^{-\lambda t}\, dE_N(\lambda).$$

The operator W_t has already been defined and studied.[1, 2] So far as V_t is concerned, we have the following theorem:

THEOREM 2. *For $t > 0$ the operator V_t satisfies the heat equation $\partial V_t/\partial t = -\Delta_M V_t$ and, given $\varphi \in H$, $V_t\varphi$ converges to φ as t approaches zero, to $F_n\varphi$ as t approaches $+\infty$. Moreover, V_t commutes with d and with δ_c.*

From Theorem 2 we may derive Theorem 3.

THEOREM 3. *Given a closed form $\varphi \in H^r$ (that is, $\varphi \in d, d\varphi = 0$), $F_n\varphi$ has the same period as φ on every r-cycle with compact support. If $\varphi \in d_c$ and $d_c\varphi = 0$, then $F_t\varphi$ has the same period as φ on every r-cycle with compact support.*

4. *Finite Manifolds.*—A subdomain (open subset) of a manifold R will be called a "bounded manifold" if its closure is compact relative to R. A bounded manifold will be called a "finite manifold" if its boundary is a manifold (not necessarily connected) of dimension $n - 1$ (n is the dimension of R) which is regularly imbedded in R and which consists of finitely many components. If R is Riemannian, we say the bounded manifold or the finite manifold is Riemannian.

THEOREM 4. *On a finite Riemannian manifold M the operators L_s and M_s are completely continuous.*

The proof of Theorem 4 is complicated; it is based on an approximation by completely continuous operators, the convergence depending on the Poincaré-Fredholm method of integral equations.

It follows from Theorem 4 that we can define completely continuous operators N_0, L_0, and M_0 satisfying, for $\varphi \in H$,

$$\begin{aligned}
&(1'') \quad \Delta_N N_0\varphi = \varphi - F\varphi, \qquad FN_0 = 0;\\
&(2'') \quad \Delta_L L_0\varphi = \varphi - F_t\varphi, \qquad F_tL_0 = 0, \qquad \delta L_0 = L_0\delta, \qquad d_cL_0 = L_0d_c;\\
&(3'') \quad \Delta_M M_0\varphi = \varphi - F_n\varphi, \qquad F_nM_0 = 0, \qquad \delta_cM_0 = M_0\delta_c, \qquad dM_0 = M_0d.
\end{aligned}$$

The existence of N_0 is based on the complete continuity of $N_s - (1/s)F$, which follows from formula (2.2). In particular, given $\varphi \in d$,

$$\varphi = d\delta_c M_0\varphi + {}_cM_0d\varphi + F_n\varphi. \tag{4.1}$$

If, in addition, $\varphi \in \delta_c$, then

$$\varphi = dM_0\delta_c\varphi + \delta_cM_0d\varphi + F_n\varphi. \tag{4.2}$$

Hence the qth cohomology group of the manifold M with real coefficients, namely $H^q(M; R)$, is isomorphic to the d-cohomology of M and hence isomorphic to $F_n{}^q$. Dually, the qth relative cohomology group modulo the boundary, namely $H^q(M, bM; R)$, is isomorphic to the d_c-cohomology of L, and hence isomorphic to $F_t{}^q$, and, since the duality operator $*$ maps $F_t{}^{n-q}$ isomorphically onto $F_n{}^q$, we have the Lefschetz duality theorem:

$$H^q(M; R) \approx H^{n-q}(M, bM; R). \tag{4.3}$$

This result is essentially the same as the theorem proved by Duff and Spencer.[5]

The boundary of M is made up of components $C_1, \ldots, C_r$, where each C_j is a compact connected differentiable manifold. If $R_1, \ldots, {}_s$ is the subspace of forms in F with vanishing tangential component on $C_1 \cup \ldots \cup C_s$ and vanishing normal component on $C_{s+1} \cup \ldots \cup C_r$, while $R_{s+1}, \ldots, {}_r$ is the set of harmonic fields with vanishing normal component on $C_1 \cup \ldots \cup C_s$ and vanishing tangential component on $C_{s+1} \cup \ldots \cup C_r$, then one may show, as above, that

$$\begin{aligned}
H^q(M, C_1 \cup \ldots \cup C_s; R) &\approx R_1, \ldots, {}_s{}^q,\\
H^q(M, C_{s+1} \cup \ldots \cup C_r; R) &\approx R_{s+1}, \ldots, {}_r{}^q,
\end{aligned}$$

and, since

$$*R_{1, \ldots, s}{}^{q} = R_{s+1, \ldots, r}{}^{n-q},$$

we have the further duality result

$$H^q(M; C_1 \cup \ldots \cup C_s; R) \approx H^{n-q}(M; C_{s+1} \cup \ldots \cup C_r; R). \tag{4.4}$$

* This work was carried out while the author was a National Science Foundation predoctoral fellow. The author expresses his appreciation to D. C. Spencer for suggesting this problem and for his help and advice in the preparation of this paper.

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[3] D. C. Spencer, "Dirichlet's Principle on Manifolds," in volume in commemoration of Professor R. von Mises (to appear).

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*ERGODIC PROPERTY OF THE BROWNIAN MOTION PROCESS**

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Let $X(t)$, $0 \leq t < \infty$, denote a separable Brownian motion (Wiener) process on the line: $X(0) \equiv 0$, $X(t)$ is continuous for all t with probability 1, and for any $t_0 < t_1 < \ldots < t_n$ the random variables $X(t_j) - X(t_{j-1})$, $j = 1, \ldots, n$, are independent and normally distributed with zero means and variances $t_j - t_{j-1}$. Kallianpur and Robbins[1] considered ratios of the form $\int_0^T f(X(t))\,dt / \int_0^T g(X(t))\,dt$ as $T \to \infty$ and proved that if $f(x)$ and $g(x)$ are bounded and summable with $\bar{f} = \int_{-\infty}^{\infty} f(x)\,dx$, $\bar{g} = \int_{-\infty}^{\infty} g(x)\,dx \neq 0$, then the above ratio tends in probability to $\bar{f}/\bar{g}$. (They also proved a similar result for the two-dimensional Brownian motion process.) We shall consider here the question of whether the limit exists with probability 1. Robbins[2] stimulated interest in this type of problem in considering the equidistribution of sums of independent random variables and of certain stochastic processes, including the Wiener process. Harris and Robbins,[3] utilizing ergodic theory of spaces with infinite measures, obtained results like those of the present note for recurrent Markov chains with discrete time parameter. Chung[4] proved, independently, the results of Harris and Robbins for the special case of Markov chains with denumerably many states. The proof used in this note will be a generalization of the one used by Chung, which in turn is based on an idea of Doeblin.[5]

Let τ be the random variable equal to $\min \{t : X(t) = 1\}$, and let $\mu(A, \tau)$ denote the Lebesgue measure of the set $\{t : X(t) \in A,\ t \leq \tau\}$, where A is any Borel set. (Since $X(t)$ is measurable with respect to the product space $\Omega \times T$, where Ω is the space of all sample functions $X(t)$ and T is the set of nonnegative real numbers t,

the set $\{t:X(t) \in A, t \leq \tau\}$ is Lebesgue measurable). Let $\mu(A) = E\mu(A, \tau)$. We now prove Lemma 1.

LEMMA 1. *If A is any bounded Borel set, then*

$$\mu(A) = 2\int_{A \cap \{x \leq 0\}} dx + 2\int_{A \cap \{0 < x \leq 1\}} (1 - x)\, dx.$$

Proof: Let $Y(t) = 1$ if $X(t) \in A$, and $t \leq \tau$ and $Y(t) = 0$ otherwise. It is easily verified that $Y(t)$ is measurable with respect to the product space $\Omega \times T$. Then we have

$$\mu(A, \tau) = \int_0^\infty Y(t)\, dt. \tag{1}$$

On taking expected values, where P denotes the probability measure over Ω, we have

$$\begin{aligned} \mu(A) &= E\int_0^\infty Y(t)\, dt = \int_\Omega \int_0^\infty Y(t)\, dt\, dP \\ &= \int_0^\infty \int_\Omega Y(t)\, dP\, dt \\ &= \int_0^\infty Pr\{X(t) \in A, t \leq \tau\}\, dt \\ &= \int_0^\infty \int_A w(x, t)\, dx\, dt = \int_A \int_0^\infty w(x, t)\, dt\, dx, \end{aligned} \tag{2}$$

where $w(x, t)$ is the density function in x of the event that simultaneously $X(t) = x$ and $t \leq \tau$. The various interchanges of integrals carried out in (2) are justified, since the integrand is always nonnegative. By applying the principle of symmetry,[6] it is seen that

$$w(x, t) = \frac{1}{\sqrt{2\pi t}} (e^{-x^2/2t} - e^{-(x - 2)^2/2t}) \qquad \text{for } x \leq 1 \tag{3}$$

and zero otherwise. It can be shown by differentiating under the integral sign or by direct evaluation that

$$\int_0^\infty w(x, t)\, dt = \begin{cases} 2 & \text{for } x \leq 0, \\ 2(1 - x) & \text{for } 0 < x \leq 1, \end{cases} \tag{4}$$

and zero otherwise. The lemma follows from (2) and (4).

Let $\tau^* = \min\{t > \tau : X(t) = 0\}$ and $\mu^*(A) = E\mu(A, \tau^*)$, where $\mu(A, \tau^*)$ is the Lebesgue measure of the set $\{t:X(t) \in A, t \leq \tau^*\}$. It follows from Lemma 1, symmetry and the independent increments property of the Brownian motion process that

$$\mu^*(A) = 2m(A), \tag{5}$$

where m is the Lebesgue measure.

LEMMA 2. *If $f(x)$ is any real-valued Borel measurable function, then*

$$E\int_0^{\tau^*} f(X(t))\, dt = 2\bar{f}. \tag{6}$$

Proof: Since $f(x)$ is Borel measurable, $f(X(t))$ is Borel measurable with probability 1, and therefore it makes sense to speak of the integral (6). Let A be any Borel set, and suppose $f(x) = 1$ if $x \in A$, and zero otherwise. Then, from (5), we have

$$E\int_0^{\tau^*} f(X(t))\, dt = 2m(A). \tag{7}$$

Lemma 2 follows by the usual approximation procedures.

THEOREM. *If $f(x)$ and $g(x)$ are any two real-valued Borel measurable functions, summable over the real line $-\infty < x < \infty$, and if $\bar{g} \neq 0$, then with probability* 1

$$\lim_{T \to \infty} \frac{\int_0^T f(X(t))\, dt}{\int_0^T g(X(t))\, dt} = \frac{\bar{f}}{\bar{g}}.$$

Proof: Consider first the case where $f(x) \leq 0$ for all x. Let $t_1 = \min \{t : X(t) = 1\}$, $t_2 = \min \{t > t_1 : X(t) = 0\}$, $t_3 = \min \{t > t_2 : X(t) = 1\}$, $t_4 = \min \{t > t_3 : X(t) = 0\}$, ... Since the probability is 1 that τ is finite, it follows that, with probability 1, the sequence $\{t_i\}$ is denumerable. Let $Z_i = \int_{t_{2i-2}}^{t_{2i}} f(X(t))\, dt$. Then the random variables Z_i are independent and identically distributed. Let $K(T)$ be the largest even number such that $t_{K(T)} \leq T$. Then

$$\sum_{i=1}^{K(T)} Z_i \leq \int_0^T f(X(t))\, dt \leq \sum_{i=1}^{K(T)+1} Z_i \tag{8}$$

Since $\Pr \{\lim_{T \to \infty} K(T) = \infty\} = 1$, it follows from the strong law of large numbers, and (6) that

$$\Pr \left\{ \lim_{T \to \infty} \frac{\int_0^T f(X(t))\, dt}{K(T)} = 2\bar{f} \right\} = 1. \tag{9}$$

If $f(x) \leq 0$, the same argument holds. Thus (9) holds for functions taking on both positive and negative values, since any function can be represented as the sum of two functions each of constant sign. The same argument holds for $g(x)$. Hence, by taking ratios (provided that $\bar{g} \neq 0$), the theorem follows.

It seems likely that the theorem proved by Kallianpur and Robbins[1] for the two dimensional Brownian motion process also holds with probability 1, but the method of proof used here does not seem to carry over. The method will, however, carry over to strongly continuous separable Markov processes having the property that the first passage time between any two distinct points is finite with probability 1 and that $\mu^*(A)$ is finite for all bounded Borel measurable sets. If the process $X(t)$ is such that for any A there exist numbers T and T' for which

$$\inf_{x \in A \cap \{x \leq 1\}} \Pr \{X(T) \leq 1 \mid X(0) = x\} > 0$$

and

$$\inf_{x \in A \cap \{x > 1\}} \Pr \{X(T') \leq 0 \mid X(0) = x\} > 0$$

then $\mu^*(A)$ will be finite.

It is interesting to note that (5) has its counterpart in the symmetrical random walk. It follows from a theorem proved by the author[7] and was pointed out by Chung[4] that the expected number of times any given state is visited before returning to the origin is equal to one.

I am indebted to Professor Herbert Robbins for calling my attention to this problem and for his interest shown during many helpful conversations.

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ON THE POSSIBILITY OF ELECTROMAGNETIC SURFACE WAVES

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1. *Introduction.*—The question of electromagnetic surface waves propagating along a plane of discontinuity between two different media was the subject of some diversity of opinion. It is related to the equally unsettled question about the merits of the conflicting solutions given by A. Sommerfeld[1] and H. Weyl[2] for the field of an electric dipole oscillating at the surface of a plane earth. A few remarks about this relation will be made in section 5, but its complete discussion will be postponed until a later date, and the present paper will be devoted to the analysis of the *independent surface wave.* It will be well, therefore, to define what is meant by this term. It is a matter of common knowledge that an ordinary plane wave, falling under an angle on a surface of discontinuity and suffering total reflection in the process, produces in the second medium an inhomogeneous wave with propagation parallel to the surface of discontinuity. But this may be called a *dependent* surface wave because it is only an adjunct of the space waves in the first medium and would not exist without them. On the other hand, an *independent* surface wave is one which consists of two inhomogeneous waves—one in each medium—running along the dividing surface. Its intensity is appreciable only in a thin layer along the surface of discontinuity and decreases exponentially to both sides.

On the whole, the results of the analysis presented in the following sections are negative: an independent surface wave is supported only by a medium with peculiar properties that are hardly available in nature (sec. 4); therefore, it is only of theoretical interest. The whole investigation hinges on the consideration of the physical constants of the materials involved. Let the complex dielectric constants of the first and second medium be noted, respectively, by

$$k^2 = \kappa = \epsilon + i\sigma, \qquad k'^2 = \kappa' = \epsilon' + i\sigma'. \tag{1}$$

In every known medium the constant σ (which is proportional to the conductivity) is positive, and this fact has a decisive influence on the results. Moreover, it is arbitrary which medium is chosen as the primed; therefore, the convention

$$\sigma' > \sigma > 0 \tag{2}$$

does not involve any loss of generality.

In most substances the real part, ϵ, is also positive; however there are exceptions like metals with $\epsilon < 0$. In order not to restrict the generality, we shall leave the sign of ϵ, ϵ' open. On the other hand, the consideration of materials with magnetic properties does not add much of interest to the analysis and only renders it cumbersome. Therefore, we shall restrict ourselves to nonmagnetic media.

If a Cartesian system of co-ordinates x, y, z is used, so that the (x, y)-plane is the plane of discontinuity and the y-axis is the direction of propagation of the surface wave, the first or *unprimed* medium, κ, will be chosen so as to fill the upper half-space ($z > 0$), and the second or *primed* medium, κ', the lower half-space ($z < 0$). It will be shown in the next section that the space dependence of the field components in the two media is, respectively, expressed by the two factors

$$F = \exp\,[i\alpha y - (\alpha^2 - k^2)^{1/2} z], \tag{3}$$

$$F' = \exp\,[i\alpha y + (\alpha^2 - k'^2)^{1/2} z], \tag{4}$$

omitting the time factor $\exp(-i\omega t)$, where ω denotes the angular frequency and α is the same in both functions.

The mathematical condition on which the existence of the surface wave depends is then (sec. 2)

$$N(\alpha^2) = k'^2(\alpha^2 - k^2)^{1/2} + k^2(\alpha^2 - k'^2)^{1/2} = 0; \tag{5}$$

this equation is derived with the help of expressions (3), (4), and the square roots in it must have the same properties as the square roots in F and F'.

A surface wave can exist if equation (5) admits of a solution subject to the physical conditions of the problem. These conditions must apply to any value of α having a physical reality and in particular to the root of equation (5) if it exists. We designate the root by α_0 and abbreviate:

$$\beta_0 = \gamma_0 + i\delta_0 = \alpha_0^2. \tag{6}$$

The conditions which α_0 and β_0 must satisfy are then as follows:

1. Since the positive y-direction is taken as the sense of propagation and the time factor is chosen with the negative exponent, $-i\omega t$, both the real and the imaginary part of α_0 must be positive (i.e., α_0 lies in the first quadrant):

$$\mathrm{Re}\,(\alpha_0) > 0, \qquad \mathrm{Im}\,(\alpha_0) > 0. \tag{7}$$

2. Condition of finiteness: the factors F, F' should not grow into infinity as z becomes infinite (positively, for $z > 0$; negatively, for $z < 0$). This requires

$$\mathrm{Re}\,(\beta_0 - \kappa)^{1/2} > 0, \qquad \mathrm{Re}\,(\beta_0' - \kappa')^{1/2} > 0. \tag{8}$$

3. Radiation condition: the factors F, F' should not contain any part representing waves moving (in the z-direction) toward the plane of discontinuity. Hence

$$\mathrm{Im}\,(\beta_0 - \kappa)^{1/2} \leq 0, \qquad \mathrm{Im}\,(\beta_0' - \kappa')^{1/2} \leq 0. \tag{9}$$

Thus the two square roots are quantities of the fourth quadrant.

When ϵ, ϵ' are both positive, the problem becomes trivial because it is easily seen that the real part of $N(\alpha_0^2)$ is then always positive, so that equation (5) cannot be satisfied and the independent surface wave cannot exist. However, we do not wish to restrict ourselves to this special case; besides, related problems may arise in which not all of conditions (7), (8), and (9) must be met. We think it, therefore, worth while to investigate first whether equation (5) admits of a solution if requirement (8) alone is postulated. This investigation is carried out in sections 3 and 4. It is found that the answer depends on the signs of the following combinations of the physical constants

$$\left.\begin{aligned} R &= (\sigma^2 - \epsilon^2)(\epsilon + \epsilon') - 2\epsilon\sigma(\sigma + \sigma'), \\ I &= -(\sigma^2 - \epsilon^2)(\sigma + \sigma') - 2\epsilon\sigma(\epsilon + \epsilon'), \end{aligned}\right\} \tag{10}$$

and R', I', which are obtained by cyclic substitution (i.e., by interchange of primed and unprimed symbols).

The results are summarized in Table 1. The structure of the functions I, I' is such that, in view of equations (2), in all cases $I' < I$; therefore, the table contains all possible sign combinations. The entries in the last column are the answers to the question: "Does a root of equation (5), subject to conditions (8), exist?" Whenever the result is "Yes," it is contingent on additional conditions that must be satisfied by the constants; they will be enumerated in section 3. In every case when a root $\beta_0 = \alpha_0^2$ exists, α_0 can be chosen so that conditions (7) are also satisfied. It is different with conditions (9), which impose additional restrictions, making it impossible to obtain a solution. Indeed, to fulfil conditions (9), it would be necessary to have $I < 0$, $I' < 0$, but these inequalities are not true in any of the cases of Table 1, where the answer is "Yes." Hence, $N(\alpha^2)$ does not possess a root satisfying the full set of conditions (7), (8), and (9), and this means that an independent surface wave cannot exist.

TABLE 1

No.	R	R'	I	I'	Result
1	Pos.	Pos.	Pos.	Pos.	Yes
2			Pos.	Neg.	Yes
3			Neg.	Neg.	No
4	Pos.	Neg.	Pos.	Pos.	Yes
5			Pos.	Neg.	No
6			Neg.	Neg.	No
7	Neg.	Pos.	Pos.	Pos.	Yes
8			Pos.	Neg.	Yes
9			Neg.	Neg.	No
10	Neg.	Neg.	Pos.	Pos.	Yes
11			Pos.	Neg.	Yes
12			Neg.	Neg.	No

In this form the conclusions apply only to finite I, I'; the limiting case when these functions vanish will be discussed in section 4.

2. *The Electric Field of the Surface Wave.*—The electric field vector $\boldsymbol{E}$ in the first medium ($z > 0$) and the magnetic, $\boldsymbol{H}$, must satisfy the equations

$$\left.\begin{aligned} \nabla^2\boldsymbol{E} + k^2\boldsymbol{E} = 0, \qquad \nabla\cdot\boldsymbol{E} = 0, \\ \boldsymbol{H} = -\left(\frac{c}{\mu\omega}\right)\nabla \times \mathbf{E}, \end{aligned}\right\} \tag{11}$$

where c denotes the velocity of light in vacuum and μ the magnetic permeability. However, we shall deal only with nonmagnetic media, where $\mu = \mu' = 1$.

It is well known and trivial that the state of polarization in which $\boldsymbol{E}$ is parallel to the surface of discontinuity does not permit of a surface wave. Hence the only interesting case is that when E_x vanishes. The components different from zero are then, with the notations of equation (4),

$$\left.\begin{aligned} E_z &= AF, \qquad E_y = -i(\alpha^2 - k^2)\alpha^{-1}AF, \\ H_x &= (ck^2/\omega\alpha)\, AF, \end{aligned}\right\} \tag{12}$$

where A is an arbitrary constant.

In the second (primed) medium ($z < 0$) analogous equations obtain, with the difference that the components and constants are primed:

$$\left.\begin{aligned} E_z' &= A'F', \qquad E_y' = i(\alpha^2 - k'^2)\alpha^{-1}A'F', \\ (H_x' &= ck'^2/\omega\alpha)A'F'. \end{aligned}\right\} \tag{12'}$$

The parameter α is the same in both cases, or else the boundary conditions could not be satisfied, which are (for $z = 0$)

$$E_y = E_y', \quad H_x = H_x'.$$

The conditions immediately lead to the so-called *determinant equation* (5), which we shall write in the form

$$N(\beta_0) = 0, \tag{13}$$

where

$$N(\beta) = \kappa'(\beta - \kappa)^{1/2} + \kappa(\beta - \kappa')^{1/2}. \tag{14}$$

The equation can be formerly resolved by transferring the second term to the right side and squaring both sides. The result is

$$\beta_0 = \frac{\kappa\kappa'}{\kappa + \kappa'}, \tag{15}$$

but it must be borne in mind that the same expression would follow if $N(\beta)$ were defined as the difference, instead of the sum, of the two terms in formula (14). Hence equation (15) is but a formal expression of the root, provided it exists, but it does not guarantee its existence. This can be put in a different way, if the complex plane of the variable β is considered: the point β_0 in the plane defined by equation (15) may or may not be the root of equation (13). The function $N(\beta)$ defines a two-sheeted Riemann surface for the complex variable β, and conditions (8) impose restrictions upon the region in this surface where the point β_0 can be located. The situation becomes very simple if we choose the branch cuts (starting at the two branch points κ and κ') as indicated in Figure 1, namely, going into infinity in the negative direction at constant imaginary height. Then all points of the first sheet of the Riemann surface are characterized by $-\pi < \varphi < \pi$ $-\pi < \varphi' < \pi$ and satisfy conditions (8), while all points of the second sheet violate them and must be excluded. If the first sheet is chosen as the physically permissible region, equations (8) are taken care of.

3. *Roots of the Determinant Equation.*—We shall now evaluate the complex quantities

$$\beta_0 - \kappa = \rho \exp i\varphi, \qquad \beta_0 - \kappa' = \rho' \exp i\varphi', \tag{16}$$

regardless of whether the point β_0 is the root of equation (13) or not. Equations (1) and (2) give, with notations (10),

$$\beta_0 - \kappa = \frac{R + iI}{Q}, \tag{17}$$

where Q is the essentially positive function

$$Q = (\epsilon + \epsilon')^2 + (\sigma + \sigma')^2, \tag{18}$$

whence follows

$$\rho = (\epsilon^2 + \sigma^2)Q^{-1/2}. \tag{19}$$

It is important to notice that the real and imaginary parts of β_0 are expressed by

$$\left.\begin{aligned} \gamma_0 &= \epsilon + \frac{R}{Q} = \frac{\epsilon'(\sigma^2 + \epsilon^2) + \epsilon(\sigma'^2 + \epsilon'^2)}{Q}. \\ \delta_0 &= \sigma + \frac{I}{Q} = \frac{\sigma'(\sigma^2 + \epsilon^2) + \sigma(\sigma'^2 + \epsilon'^2)}{Q}. \end{aligned}\right\} \tag{20}$$

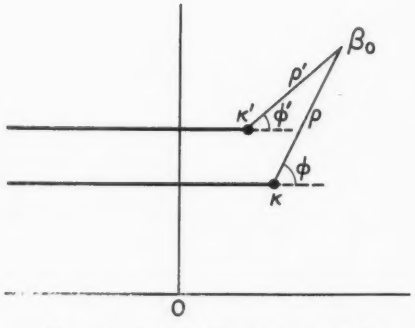

FIG. 1.—Complex plane of the variable β

Because of conditions (2), δ_0 is always positive, and this means that the point $\beta_0 = \alpha_0^2$ always lies in the positive complex half-plane (first or second quadrant). Hence it is always possible to choose the sign of $\alpha_0 = \pm\beta_0^{1/2}$ in such a way as to satisfy conditions (7).

The difference $\beta_0 - \kappa'$ is expressed by equations quite similar to (17) and (19), which are at once obtained by interchanging the primed and unprimed symbols in the above expressions.

We turn now to the quantities directly entering into the function $N(\beta_0)$. Obviously,

$$(\beta_0 - \kappa)^{1/2} = \rho^{1/2} \exp {}^1/_2 i\varphi, \qquad (\beta_0 - \kappa')^{1/2} = \rho'^{1/2} \exp {}^1/_2 i\varphi'. \tag{21}$$

The complex constants κ, κ' can be expressed in terms of these quantities. Indeed, by eliminating β_0 and κ' from equations (15) and (16), we find

$$(\kappa + \rho \exp i\varphi)^2 = \rho\rho' \exp i(\varphi + \varphi'),$$

whence

$$\left.\begin{aligned} \kappa &= \pm(\rho\rho')^{1/2} \exp {}^1/_2 i(\varphi + \varphi') - \rho \exp i\varphi, \\ \kappa' &= \pm(\rho\rho')^{1/2} \exp {}^1/_2 i(\varphi + \varphi') - \rho' \exp i\varphi'. \end{aligned}\right\} \tag{22}$$

Substituting in expression (14) for $N(\beta_0)$,

$$N(\beta_0) = (\pm 1 - 1)\,(\rho\rho')^{1/2} \exp {}^1/_2 i(\varphi + \varphi') \times [\rho^{1/2} \exp {}^1/_2 i\varphi + \rho'^{1/2} \exp {}^1/_2 i\varphi'].$$

It is apparent that only the *upper sign* leads to $N(\beta_0) = 0$ and guarantees that β_0 is a root of the determinant equation. The problem is, therefore, to find out which sign in equations (22) is the correct one under the various circumstances. In view of relations (16), these equations can be thrown into the form

$$\pm(\rho\rho')^{1/2} \exp {}^1/_2(\varphi + \varphi') = \beta_0. \tag{23}$$

Both sides of this equation are reducible to the functions R, R', I, I'; it is, therefore, a routine matter to determine in every particular case which sign is correct. It will be sufficient to illustrate the simple procedure by two examples. (The numbering of them will be that of Table 1.)

1. In this case $0 < \varphi < {}^1/_2\pi$, $0 < \varphi' < {}^1/_2\pi$, whence $0 < {}^1/_2(\varphi + \varphi') < {}^1/_2\pi$, and

$$\cos {}^1/_2(\varphi + \varphi') > 0, \qquad \sin {}^1/_2(\varphi + \varphi') > 0.$$

On the other hand, it is apparent from equations (20) that the point β_0 lies in its complex plane above and to the right of both branch points κ and κ'. Therefore, equation (23) will have the upper sign if β_0 lies in the positive quadrant of the upper half-plane (first quadrant). Since δ_0 is always positive, the only additional requirement is

$$\gamma_0 > 0. \tag{24}$$

2. Here $0 < \varphi < {}^1/_2\pi$, $-{}^1/_2\pi < \varphi' < 0$, whence ${}^1/_4\pi < {}^1/_2(\varphi + \varphi') < {}^1/_4\pi$. The point β_0 lies horizontally to the right of both branch points κ, κ' and vertically between their heights. Two cases must be distinguished:

a) $0 < {}^1/_2(\varphi + \varphi') < \pi/4$ (an additional condition), $\cos {}^1/_2(\varphi + \varphi') > 0$, $\sin {}^1/_2(\varphi + \varphi') > 0$. Again the upper sign is obtained, if requirement (24) is satisfied.

b) $-\pi/4 < {}^1/_2(\varphi + \varphi') < 0$; then $\cos {}^1/_2(\varphi + \varphi') > 0$, $\sin {}^1/_2(\varphi + \varphi') < 0$. Since δ_0 is always positive, the upper sign cannot be obtained.

We list the additional conditions necessary to achieve the upper sign in the cases of Table 1 where the answer is "Yes":

4*a* and 7*a*. When ${}^1/_2\pi < (\varphi + \varphi') < \pi$, $\gamma_0 > 0$, $\epsilon' > \epsilon$, $\epsilon' > 0$.

4*b* and 7*b*. When $\pi < (\varphi + \varphi') < 3\pi/2$, $\gamma_0 < 0$, $\epsilon' > \epsilon$, $\epsilon < 0$.

8. $\gamma_0 > 0$, $\epsilon > \epsilon'$, $\epsilon > 0$.

10. $\gamma_0 < 0$.

11*a*. When $0 < (\varphi + \varphi') < {}^1/_2\pi$, $\gamma_0 > 0$, $\epsilon > 0$, $\epsilon' > 0$.

11*b*. When $-{}^1/_2\pi < (\varphi + \varphi') < 0$, $\gamma_0 < 0$.

The conclusions from these results were drawn in section 1.

4. *Special Case of Nonconducting Media.*—Our general analysis includes also the case that either of the media, or both, are nonconducting, inasmuch as we can consider vanishing conductivity as the limiting case of very small σ, σ'. The case when only one of these parameters vanishes is in no way different from the general and needs no separate discussion, because β_0 remains complex. But it is well to say a word about the case $\sigma = \sigma' = 0$. According to equation (10), there follows,

$$I = I' = 0,$$

$$R = -\epsilon^2(\epsilon + \epsilon'), \qquad R' = -\epsilon'^2(\epsilon + \epsilon'),$$

so that two cases must be distinguished.

a) When $(\epsilon + \epsilon') > 0$, both R and R' are negative, and hence $\gamma_0 < \epsilon$ and $\gamma_0 < \epsilon'$. Therefore, the factors of z in equation (4) reduce to $(\gamma_0 - \epsilon)^{1/2}$, $(\gamma_0 - \epsilon')^{1/2}$ and are purely imaginary. This means that the amplitudes of the waves do not decrease away from $z = 0$. The field does not represent a surface wave but two space waves, each filling the whole of one of the two half-spaces. Consequently, this case is of no interest for our problem.

b) When $(\epsilon + \epsilon') < 0$, there follows $R > 0$, $R' > 0$ and hence $\gamma_0 > \epsilon$, $\gamma_0 > \epsilon'$. The factors $(\gamma_0 - \epsilon)^{1/2}$, $(\gamma_0 - \epsilon')^{1/2}$ are both real, so that conditions (9) are satisfied. This case falls under Nos. 1, 2, and 3 of Table 1, with the difference that I, I' go to the limit $I = I' = 0$ as follows:

$$I = \lim [\epsilon^2(\sigma + \sigma') + 2\epsilon\sigma|\epsilon + \epsilon|],$$

$$I' = \lim [\epsilon'^2(\sigma + \sigma') + 2\epsilon'\sigma'|\epsilon + \epsilon'|].$$

Nos. 1 and 2 require the validity of the additional condition (24), which reduces to

$$\epsilon\epsilon'(\epsilon + \epsilon') > 0.$$

It is clear that the requirements of No. 2 can be satisfied by letting $\epsilon > 0$, $\epsilon' < 0$, and by appropriately adjusting the ratios of all the parameters.

This is the only case in which an independent surface wave can exist, but even this case is purely theoretical and without any practical significance. Apart from the fact that a nonconducting medium with a negative dielectric constant is not readily available, every known material of nature possesses a vestigial conductivity, and the slightest positive values of σ, σ' would invalidate the argument by bringing into play conditions (9).

5. *Bearing on Other Problems.*—The best-known example of a related problem is the propagation of waves emitted by a radio antenna along the surface of the earth, especially in the form given to it by Sommerfeld, as mentioned in the Introduction. The perturbation wave due to the presence of the earth's surface is set up in the form of an integral whose integrand is analogous in type to our expressions (4). However, the integral is extended over *real* values of α, and, in so far as subsidiary conditions of the character of (7), (8), and (9) are necessary, they are required prima facie for real values of α. For the purposes of evaluation the path of integration is then deformed into a contour integral in the complex plane, and the equation $N(\alpha^2) = 0$, identical with our equation (5), becomes important because its root determines the pole of the integrand. The question whether this equation has a root is analogous with the one treated above in section 3, but the writer is not prepared to say that it is identical with it. Indeed, the complex root point has no immediate physical meaning, and it is not obvious that conditions (7), (8), and (9) must be satisfied for it. The problem needs additional analysis and is now being studied.

As the end result of his evaluation, Sommerfeld found two perturbation waves: (1) a space wave with an intensity decreasing with the inverse square of the distance from the antenna and (2) a surface wave propagating along the earth and decreasing with the inverse first power. Such a situation seems questionable[3] in the light of our results stated above; it may be argued that the surface wave outlasts the space wave and becomes independent at large distances, in contradiction to the result that an independent surface wave cannot exist. However, even this conclusion needs further investigation; it is not entirely cogent, because the amplitudes of both waves also contain exponentially decreasing factors, a fact which limits the independence of the surface wave.

[1] A. Sommerfeld, *Ann. Physik*, **28**, 665, 1909.

[2] H. Weyl, *Ann. Physik*, **60**, 481, 1919.

[3] Questions were raised by the writer in these PROCEEDINGS, **33**, 195, 1947; the point of view expressed there is now only partially maintained by him.

ERRATA: OVARIAN HORMONES AND THE IONIC BALANCE OF UTERINE MUSCLE

In the article of the foregoing title appearing in these PROCEEDINGS, **40**, 515–521, 1954, the following corrections should be made:

P. 518, Table 2: Under the headings "Estrogen*" and "Progesterone" read "CV_x" instead of "CV_{sx}"; the note to Table 2 reads correctly: "* x = arithmetic mean Na, K, and ECW, respectively, of samples from one and the same animal; n = number of animals; $\bar{x}$ = arithmetic mean, $\Sigma x/n$; s_x = standard deviation: $\sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$; CV_x = coefficient of variation: $\frac{100\, s_x}{\bar{x}}$."

P. 519, line 12: Read "the difference" instead of "and difference."

P. 519, line 20: Read "is not significantly different" instead of "is significantly different."

P. 520, line 29: Read "...Estrogen: Na $\geq$ 45.6; K $\leq$ 128; progesterone: Na $\leq$ 24.1; K $\geq$ 150."

BENI HORVATH

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